1.1 TURBOMACHINE

While discussing the minimal number of components needed to constitute a heat engine [16], it was mentioned that mechanical energy output is obtained from an expander (work output device), whereas mechanical energy input to the system is due to a pump or a compressor which raises the pressure of the working fluid, a liquid or a gas. Both the expander and the pump (or compressor), are devices which provide work output or accept work input to affect a change in the stagnation state (Sec. 1.3) of a fluid. These devices are often encountered as parts of heat engines, though they can function independently as well. The principles of operation of both a work output device (e.g., an internal combustion engine of the reciprocating type) and a work input device (the reciprocating air-compressor [16]), have already been studied. In addition to these two types, there exist other devices which are invariably of the rotary\(^1\) type where energy transfer is brought about by dynamic action, without an impervious boundary that prevents the free flow of a fluid at any time. Such devices are called turbomachines.

The turbomachine is used in several applications, the primary ones being electrical power generation, aircraft propulsion and vehicular propulsion for civilian and military use. The units used in power generation are steam, gas and hydraulic turbines, ranging in capacity from a few kilowatts to several hundred and even thousands of megawatts, depending on the application. Here, the turbomachine drives the alternator at the appropriate speed to produce power of the right frequency. In aircraft and heavy vehicular propulsion for military use, the primary driving element has been the gas turbine. The details of these types of machines will be provided in later chapters.

The turbomachine has been defined differently by different authors, though these definitions are similar and nearly equivalent. According to Daily [1], the turbomachine is a device in which energy exchange is accomplished by hydrodynamic forces arising between a moving fluid and the rotating and stationary elements of the machine. According to Wislicenus [2], a turbomachine is characterized by dynamic energy exchange between one or several rotating elements and a rapidly moving fluid.

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\(^1\) Rotary type machines such as gear pump and screw pump are positive displacement machines and work by moving a fluid trapped in a specified volume.
Binder [3] states that a turbomachine is characterized by dynamic action between a fluid and one or more rotating elements. A definition to include the spirit of all the preceding definitions would be: A turbomachine is a device in which energy transfer occurs between a flowing fluid and a rotating element due to dynamic action resulting in a change in pressure and momentum of the fluid. Mechanical energy transfer occurs into or out of the turbomachine, usually in steady flow. Turbomachines include all those types that produce large-scale power and those that produce a head or pressure, such as centrifugal pumps and compressors.

The principal components of a turbomachine are: (i) a rotating element carrying vanes operating in a stream of fluid, (ii) a stationary element or elements which generally act as guide vanes or passages for the proper control of flow direction and the energy conversion process, (iii) an input and/or an output shaft, and (iv) a housing (Fig. 1.1). The rotating element carrying the vanes is also known by the names rotor, runner, impeller, etc., depending upon the particular application. Energy transfer occurs only due to the exchange of momentum between the flowing fluid and the rotating elements; there may not be even a specific boundary that the fluid is not permitted to cross. Details relating to these will be discussed in the following sections.

![Fig. 1.1. Schematic cross-sectional view of a turbine showing the principal parts of the turbomachine.](image-url)

The stationary element is also known by different names—among them guide-blade or nozzle—depending on the particular machine and the kind of flow occurring in it. A stationary element is not a necessary part of every turbomachine. The common ceiling fan used in many buildings in India to circulate air during summer and the table fan are examples of turbomachines with no stationary element. Such machines have only two elements of the four mentioned above: an input shaft and a rotating blade element.

Either an input or an output shaft or both may be necessary depending on the application. If the turbomachine is power-absorbing, the enthalpy of the fluid flowing through it increases due to mechanical energy input at the shaft. If the turbomachine is power-generating, mechanical energy output is obtained at the shaft due to a decrease in enthalpy of the flowing fluid. It is also possible to have power-transmitting turbomachines which simply transmit power from an input shaft to an output shaft, just like a clutch-plate gear drive in a car which transmits the power generated by the reciprocating engine to the shaft which drives the wheels. In principle,
the device acts merely as an energy transmitter to change the speed and torque on the driven member as compared with the driver. There are many examples of these types of machines. Examples of power-absorbing turbomachines are mixed-flow, axial-flow and centrifugal pumps, fans, blowers and exhausters, centrifugal and axial compressors, etc. Examples of power-generating devices are steam, gas and hydraulic turbines. The best known examples of power-transmitting turbomachines are fluid-couplings and torque-converters for power transmission used in automobiles, trucks and other industrial applications.

The housing too is not a necessary part of a turbomachine. When present, it is used to restrict the fluid flow to a given space and prevent its escape in directions other than those required for energy transfer and utilization. The housing plays no role in the energy conversion process. The turbomachine that has housing is said to be enclosed and that which has no housing is said to be extended [4]. The ceiling-fan shown in Fig. 1.2 is an example of an extended turbomachine and all the rest shown in the figure are enclosed turbomachines.

![Fig. 1.2.](image)

Turbomachines are also categorized by the direction of fluid flow as shown in Fig. 1.2. The flow directions are: (i) axial, (ii) radial and (iii) mixed. In the axial-flow and radial-flow turbomachines, the major flow directions are approximately axial and radial respectively, while in the mixed-flow machine, the flow usually enters the rotor axially and leaves radially or vice versa. Mixed flow may also involve flow over the surface of a cone. An example of a mixed-flow machine is a mixed-flow pump. A radial flow machine may also be classified into radial inward flow (centripetal) or radial outward flow (centrifugal) types depending on whether the flow is directed towards or away from the shaft axis.
In a positive-displacement machine\(^2\), the interaction between the moving part and the fluid involves a change in volume and/or a translation of the fluid confined in a given boundary. During energy transfer, fluid expansion or compression may occur in a positive-displacement machine without an appreciable movement of the mass centre-of-gravity of the confined fluid. As such, changes in macroscopic kinetic energy and momentum may be neglected in most of these machines. The movement of a piston or gear-tooth causes changes in fluid volume because of the displacement of the boundaries, i.e., the fluid cannot escape from the boundaries except due to unavoidable leakage. An expansion or contraction may occur if the fluid is compressible as for example, in a balloon being filled with air. The action is therefore nearly static and completely different from that of a turbomachine where the action is fast, dynamic and the energy transfer occurs without the necessity for a confining boundary. (In compressible flow handling machines, fluid flows at very high velocities approaching acoustic speed at certain locations).

The differences between positive-displacement machines and turbomachines are clarified further by comparing their modes of action, operation, energy transfer, mechanical features etc., in the following:

- **Action**: A positive-displacement machine creates thermodynamic and mechanical action between a nearly static fluid and a relatively slowly moving surface. It involves a change in volume or a displacement (bodily movement of the confined fluid).
  
  A turbomachine creates thermodynamic and dynamic interaction between a flowing fluid and rotating element and involves energy transfer with pressure and momentum changes. There is no positive confinement of the fluid at any point in the system.

- **Operation**: The positive-displacement machine commonly involves a reciprocating motion and unsteady flow of the fluid, though it is not impossible for the machine to have a purely rotary motion and nearly steady flow. Examples of such rotating positive-displacement machines are gear-pumps and screw-pumps. However, since the fluid containment is positive, stopping a positive-displacement machine during operation may trap a certain amount of fluid and maintain it indefinitely in a state different from that of the surroundings, if heat transfer and leakage are completely absent in a theoretical sense.

---

\(^2\) A positive-displacement machine is one which takes a fresh charge at the beginning of each cycle and discharges it at the completion of the cycle.

---

Fig. 1.3. Screw pump and gear pump.
The turbomachine involves, in principle, a steady flow of fluid and a purely rotary motion of the mechanical element. A turbomachine may also involve unsteady flow for short periods of time, especially while starting, stopping or during changes of loads. However, in most instances, the machine is designed for steady-flow operation. As there is no positive containment of the fluid, stopping of the machine will let the fluid undergo a change of state (in a matter of milliseconds), and become the same as that of the surroundings.

- **Mechanical Features:** The positive-displacement machine commonly involves rather low speeds and is relatively complex in mechanical design. It is usually heavy per unit of output and employs valves which are open only part of the time, as in reciprocating machines. Also, rather heavy foundations are usually needed because of reciprocating masses and consequent vibration problems. Generally in such machines, the mechanical features are more complex than in turbomachines.

Turbomachines usually employ high rotational speeds, are simple in design principle and are generally light in weight per unit of power output. Their foundations may be quite light since vibration problems are not severe. They do not employ valves that open and close during steady-state operation. Usually, only inexpensive associated equipment is required.

- **Efficiency of Conversion Process:** In positive-displacement machines, the use of positive containment and a nearly static energy transfer process may result in a higher efficiency relative to that of a turbomachine which employs a dynamic process including high-speed fluid flow. The higher efficiency of energy conversion used to be the advantage of a positive-displacement machine as compared with a turbomachine which used to exhibit somewhat lower efficiencies. Compression by dynamic action often involves higher losses and hence lower efficiencies, though expansion in a turbomachine results in better efficiencies than in compression. Nevertheless, both these often used to fall short of the corresponding reciprocating machine in performance. In modern turbomachines which are designed through the aid of computers with high quality, efficient software, the difference between compressive and expansive efficiencies is not large and these are both the same as the efficiencies obtainable with reciprocating machines.

- **Volumetric Efficiency:** The volumetric efficiency of a machine with positive-displacement is normally well below that of turbomachines and in some cases, very low because of the opening and the closing of valves needed for continuous operation. In turbomachines, during steady state operation, there exist no inlet and outlet valves and the volumetric efficiency differs little from 100%. Also, since the flow is continuous and the fluid velocities are high, a turbomachine has a high fluid handling capacity per kilogram weight of the machine. As an example, a 300 kW gas turbine plant typically handles about 22 kg·s⁻¹ of air and has a weight of 900 kg. Thus, the specific power output of this plant is 15 kJ·kg⁻¹ of air and it has a power plant weight per unit mass flow rate of air between 10 and 100. In comparison, an aircraft reciprocating power plant producing 300 kW handles 2 kg·s⁻¹ of air and has a weight of 1000 kg. Thus, its specific weight is 500 per kg·s⁻¹ of air flow. For all types of industrial power plants, the specific weight of a reciprocating plant is about 10-15 times that of a turbo-power plant.
Fluid Phase Change and Surging: Phase changes occurring during flow through a turbomachine can frequently cause serious difficulties to smooth operation. Examples of these are cavitation at pump inlets and hydraulic turbine outlets as well as condensation in steam turbines resulting in blade erosion and/or a deterioration of machine performance. Surging or pulsation (Chapter 6, a phenomenon associated with turbomachinery), is caused by an unstable flow situation due to a rising head-discharge characteristic. It is characterized by the pulsation of fluid pressure between the inlet and the outlet of the turbomachine, i.e., the reversal of flow direction accompanied by violent flow fluctuations. The machine may vibrate violently, and under certain operating conditions, may even be damaged by these vibrations. The performance of the device deteriorates considerably even when the flow fluctuations are not violent. Problems of phase change pulsation and surging are of no importance in positive-displacement machines.

1.3 STATIC AND STAGNATION STATES

In dealing with turbomachines, one is concerned with fluids, often compressible and moving at high speeds exceeding the speed of sound. Even in turbomachines dealing with incompressible fluids where the velocities are relatively low, the kinetic and potential energies of the fluid are often large and constitute major fractions of the total energy available for conversion into work. Simplistic approaches which neglect potential and kinetic energies cannot provide sufficiently accurate results for design. It is therefore necessary to formulate equations based on the actual state of the fluid including all the energies at the given point in the flow. Taking these factors into account, we use the equations of the First and Second laws of Thermodynamics to specify two fluid states called respectively, the ‘Static’ and the ‘Stagnation States’ which will be discussed below.

The Static State: First, consider a fluid flowing at a high speed through a duct. In order to measure the properties of the fluid, one may insert an instrument such as a pressure gauge or a thermometer at some point in the flow. One can imagine two types of measurements, one in which the measuring instrument moves at the same local speed as that of the fluid particle and another in which it is stationary with respect to the particle the properties of which are under investigation. Measurements of the first type made with an instrument which moves with the same local speed as the particle are said to determine a ‘static’ property of the fluid. Note that what is stationary is neither the fluid nor the instrument to measure the property—both of them may move except that the measuring instrument moves at the same speed as the fluid locally and is therefore at rest with respect to the particle of the fluid. For example, one can consider a pressure measurement made with a static pressure gauge which is usually fixed to the side of the duct. In this case, the fluid particle and the instrument are at rest with respect to each other at the point where the measurement is being made. Hence, the measured pressure is a static pressure. Any measurement made in consonance with this stipulation determines a static property, be it one of pressure, temperature, volume, or any other, as specified. The state of the particle fixed by a set of static properties is called the ‘Static State’.
The Stagnation or Total State\(^3\): The stagnation state is defined as the terminal state of a fictitious, isentropic, work-free and steady-flow process during which the macroscopic kinetic and potential energies of the fluid particle are reduced to zero, the initial state for the process being the static state. The macroscopic kinetic and potential energies are those measured with respect to an arbitrary and pre-specified datum state.

The stagnation state as specified above is not representative of any true state of the fluid. No real process leads to the stagnation state, because no real process is truly isentropic and perfectly free from thermal exchange with the surroundings. Despite the impossibility of achieving it, if proper care is taken to account for errors in measurement and appropriate corrections incorporated, many of the properties measured with instruments like Pitot tubes, thermocouples, etc., do provide readings that approximate stagnation properties closely. Further, stagnation property changes provide ideal values against which real machine performance can be compared. These properties and the state defined by them (the stagnation state), are thus of great importance in turbomachinery.

By using the definition of a stagnation state, it is possible to obtain expressions for stagnation properties in terms of static properties. Considering any steady-flow process, the First Law of Thermodynamics \([15]\) gives the equation:

\[
q - w = \Delta h + \Delta ke + \Delta pe, \tag{1.1}
\]

where, \(q\) and \(w\) are respectively the energy transfers as heat and work per unit mass flow, \(h\) is the static enthalpy and \(ke\) and \(pe\) are respectively the macroscopic kinetic and potential energies per unit mass. It is known that \(ke = V^2/2\), and \(pe = gz\), \(V\), being the fluid particle velocity and \(z\), the height of the particle above the datum at the point under consideration.

Since the static state is the initial state in a fictitious isentropic, work-free, steady flow process and the stagnation state is the terminal state where both the kinetic and potential energies are zero, the difference in enthalpies between the stagnation and static states is obtained by setting \(q = w = 0\), \(\Delta h = h_o - h_i\), \(ke_o = 0\) and \(pe_o = 0\) in Eq. (1.1). There is then obtained:

\[
h_o - (h_i + ke_i + pe_i) = 0 \text{ or, } h_o = (h + ke + pe), \tag{1.2}
\]

where the subscript \(o\) represents the stagnation state and \(i\) represents the initial static state. Equation (1.2) follows from the fact that at the stagnation state, both the kinetic and potential energies are zero. In the last part of Eq. (1.2), the subscript, \(i\), has been removed and from here onwards, the properties at the static state will be indicated without the subscript as shown. The enthalpy \(h_o\) in the stagnation state has thus been expressed in terms of three known properties, \(h\), \(ke\) and \(pe\) of the static state.

As stated earlier, it is necessary that the process changing the state from static to stagnation be isentropic, i.e., \(s_o = s\), and hence, the entropy in the stagnation state is equal to the entropy in the static state. Thus, two independent stagnation properties, namely the enthalpy \(h_o\) and the entropy \(s_o\), have been determined in terms of the known properties at the static state. Since

\(^3\) This definition was given by Dean R.H. Zimmerman who was Visiting Professor of Mechanical Engineering at the start of IIT-K, 1962-1967.
according to the ‘State Postulate’, the knowledge of any two independent properties at a specified state is sufficient to fix the state of a simple compressible substance [15], the stagnation state is totally determined and it should be possible to determine any other required property of the stagnation state in terms of the two known properties, \( h_o \) and \( s_o \). Also, according to the Second Law of Thermodynamics, since \( T \cdot ds = dh - v \cdot dp \), and the entropy remains constant in the change from static to stagnation state, \( ds = 0 \) and \( dh = v \cdot dp \), \( v = 1/\rho \), being the specific volume and \( \rho \), the density. Hence, integration yields for the change from static to stagnation state:

\[
h_o - h = \int dh = \int v \cdot dp.
\]  

(1.3)

The integration on the right hand side depends on the variation of volume with respect to pressure in an isentropic process. If the \( p-v \) property relation is known, the equation above may be integrated and one can determine the stagnation pressure \( p_o \) in terms of the static enthalpy \( h \) and the static pressure \( p \). This will be done for two special cases.

(a) **Incompressible Fluid**: For an incompressible fluid, \( dv = d(1/\rho) = 0 \), \( \rho \) being the density of the fluid. Since the density is constant and independent of state:

\[
h_o - h = (p_o - p)/\rho.
\]

Thus, on using Eq. (1.2),

\[
p_o/\rho = p/\rho + (h_o - h) = p/\rho + V^2/2 + gz.
\]  

(1.4)

The stagnation pressure of the incompressible fluid has now been expressed in terms of its static pressure, velocity and height above a specified datum. According to the First Law of Thermodynamics, the stagnation enthalpy \( h_o \) and the stagnation pressure \( p_o \) should be constant along any streamline which experiences no energy transfer as heat or as work. Hence, for an incompressible, frictionless fluid in steady flow, it is seen that the stagnation pressure remains a constant along a streamline in an un-accelerated coordinate system. This is the Bernoulli’s theorem studied in Fluid Mechanics.

In addition, for a change from the static to the stagnation state of an incompressible fluid, since there is no entropy change and \( p \cdot dv = 0 \), \( T \cdot ds = du + p \cdot dv \) yields \( du = 0 \). Hence, \( u = u_o \), i.e., the internal energies of an incompressible fluid in the static and stagnation states are equal. Moreover, since the internal energy of an incompressible fluid is a function of temperature alone, one concludes that:

\[
u_o - u = c(T_o - T) = 0, \ i.e., \ T_o = T.
\]  

(1.5)

The local static and stagnation temperatures are equal to each other at every point in incompressible and loss-free fluid flow.

(b) **Perfect Gas**: Since the enthalpy of a perfect gas is a function of temperature alone, from Eq. (1.2), with \( ke = V^2/2 \) and \( pe = gz \) one gets:

\[
c_p T_o = c_p T + V^2/2 + gz \ or \ T_o = T + (V^2/2 + gz)/c_p.
\]  

(1.6)

In compressible flow machines, fluid velocities vary from about 60 m.s\(^{-1}\) to 600 m.s\(^{-1}\) or more, whereas the maximum value of \( z \) is rarely in excess of 4 m in most steam and gas turbines. As such, even at the minimum flow velocity,

\[V^2/2 = 60^2/2 = 1800 \text{ J.kg}^{-1} \text{ and, } gz = (9.81)(4) = 39.24 \text{ J.kg}^{-1}.
\]
The calculations above indicate that the magnitude of kinetic energy is far in excess of the potential energy in most compressible flow machines. It is therefore usual to neglect the term $gz$ in comparison with the term $V^2/2$ and to write the equation to compute the stagnation temperature of a perfect gas in the form:

$$T_o = T + V^2/(2c_p). \quad \text{\ldots(1.7)}$$

One can now determine the stagnation temperature by using the substitution $c_p = \gamma R / (\gamma - 1)$, to obtain:

$$T_o = T[1 + (\gamma - 1)V^2/(2\gamma R T)] \quad \text{\ldots(1.8a)}$$

$$p_o = p[1 + (\gamma - 1)M^2/2], \quad \text{\ldots(1.8b)}$$

where $M$ is the local Mach number of a perfect gas defined by the equation $M = V/a$, in which the speed of sound in the gas at the static temperature $T$, is denoted by the symbol $a = (\gamma R T)^{1/2}$. Again, since for the isentropic expansion of a perfect gas, $v_o/v = (p/p_o)^{1/\gamma}$, one can write:

$$\frac{p_o}{p} = \beta^{\gamma(\gamma - 1)}, \quad \beta = 1 + (\gamma - 1)M^2/2. \quad \text{\ldots(1.9a)}$$

With this simplification in notation, the expressions for the stagnation temperature $T_o$, (Eq. 1.8a) and stagnation pressure $p_o$ (Eq. 1.9a) may be rewritten in the forms:

$$T_o = T\beta, \quad p_o = p\beta^{\gamma - 1/\gamma}. \quad \text{\ldots(1.9b)}$$

**Example 1.1.** Dry saturated steam at 1 atm. static pressure flows through a pipe with a velocity of 300 m.s⁻¹. Evaluate the stagnation (total) pressure and the stagnation temperature of the steam:

(a) By using steam tables and (b) by assuming steam to behave as a perfect gas with $\gamma = 1.3$.

**Data:** Sat. Steam flow, static pr. $p = 1.013$ bar, velocity $V = 300$ m.s⁻¹.

**Find:** The stagnation pressure $p_o$ and the stagnation temperature $T_o$.

(i) Use steam tables, (ii) Treat steam as a perfect gas with $\gamma = 1.3$.

**Solution:** In working this example and other examples in Chapters 1 and 2, the use of Steam Tables and Mollier chart will be exhibited, though it is possible to solve the problem with the help of a computer program without the use of either the tables or the chart. The procedure for writing a computer program to solve similar problems will be provided in Chapter 3.

(i) The static temperature corresponding to a saturation pressure of 1.013 bar is $T = 100^\circ$C. By referring to the steam tables for the properties of saturated steam at the temperature 100°C (Table A.2 from Appendix A), we get for the static enthalpy, $h = 2675.9$ kJ·kg⁻¹ and for the saturation static entropy $s = 7.3549$ kJ·kg⁻¹·K⁻¹. Hence, from Eq. (1.6), neglecting potential energy (since steam is a compressible substance and its potential energy is small), one gets for the stagnation enthalpy Eq. (1.7),

$$h_o = h + ke + pe = 2675.9 + 300^2/(2 \times 1000) = 2720.9 \text{ kJ·kg}^{-1}.$$

Stagnation entropy, $s_o = s$ (static entropy) = 7.3549 kJ·kg⁻¹·K⁻¹.

On referring to the Mollier chart with the values of $h_o$ and $s_o$ specified above, we get:

Total pressure, $p_o = 1.246 \text{ bar}$; Total temperature $T_o = 121^\circ$C.
(ii) If steam behaves as a perfect gas with $\gamma = 1.3$ and $R = 8317/18 = 462.06 \text{ J/kg}^{-1} \text{K}^{-1}$,

$$
T_o = T[1 + (\gamma - 1)V^2/(2\gamma R T)]
$$

$$
= 373.15 \{1 + 0.3 \times 300^2/[2(1.3)(462.06)(373.15)]\} = 395.63 \text{ K = 122.5°C.}
$$

This newly calculated value of total temperature agrees reasonably well with that calculated by using Mollier chart and is therefore satisfactory. Then, stagnation pressure:

$$
p_o = p_o(\frac{T_o}{T})^{\gamma/(\gamma - 1)} = 1.013(395.63/373.15)^{3.5} = 1.243 \text{ bar.}
$$

The newly computed pressure agrees even better with the previous value obtained by using steam tables. This is the reason that for quick calculations, we can simply use the perfect gas equations and obtain reasonably good results. The fact that superheated steam behaves nearly like a perfect gas can be used to obtain quick approximations to the properties when its pressure is well below critical. This fact will be utilized in Chapter 3 to write a computer program to calculate the states of superheated steam undergoing an isentropic expansion.

### 1.4 FIRST AND SECOND LAWS OF THERMODYNAMICS APPLIED TO TURBOMACHINES

Fluid flow in turbomachines always varies in time, though it is assumed to be steady when a constant rate of power generation occurs on an average. This is due to small load fluctuations, unsteady flow at blade-tips, the entry and the exit, separation in some regions of flow etc., which cannot be avoided, no matter how good the machine and load stabilization may be. Similar statements can be made for power absorbing turbomachines as well. Nevertheless, on an overall basis when the average over a sufficiently long time is considered, turbomachine flows may be considered as steady. This assumption permits the analysis of energy and mass transfer by using the steady-state control volume equations. Assuming further that there is a single inlet and a single outlet for the turbomachine across the sections of which the velocities, pressures, temperatures and other relevant properties are uniform, one writes the steady flow equation of the First Law of Thermodynamics in the form:

$$
Q + \dot{m} (h_1 + V_1^2/2 + gz_1) = P + \dot{m} (h_2 + V_2^2/2 + gz_2).
$$

...(1.10)

Here, $Q =$ Rate of energy transfer as heat across the boundary of the control volume,

$P =$ Power output due to the turbomachine, and

$$
\dot{m} =$ Mass flow rate.

Note: While making calculations, if enthalpy $h$ is expressed in kJ kg$^{-1}$, then both the kinetic and potential energy terms, $V^2/2$ and $gz$ in Eq. (1.10) and other similar equations should be divided by 1000 during calculations.

Since $h_o = h + V^2/2 + gz$, (Eq. 1.3), one obtains:

$$
q - w = \Delta h_o,
$$

where, $\Delta h_o = h_{o2} - h_{o1}$,

...(1.11a)
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represents the change in stagnation enthalpy between the inlet and the outlet of the turbomachine. Also, \( q = Q/\dot{m} \) and, \( w = P/\dot{m} \) represent respectively, the heat and mass transfer per unit mass flow through the control volume.

Generally, all turbomachines are well-insulated devices. Even though the fluid velocity is high and the fluid passes through the turbomachine within fractions of a millisecond in steam and gas turbines, the energy loss through the insulation is very small (usually, \( q < 0.25\% \) of \( w \)). In incompressible flow devices involving liquids like water and oil, there is no heat transfer possible since most of them operate at room temperature. Further, even when they handle warm liquids, the specific heats are so high that the liquids undergo negligible temperature changes during their passage through the pump. By neglecting \( q \) therefore, one can treat a turbomachine like a perfectly insulated device for which:

\[
\Delta h_o = -w = -P/\dot{m} \quad \text{or,} \quad dh_o = -\delta w, \quad \ldots (1.11b)
\]

where the second part of the above equations applies to a device that gives rise to infinitesimally small changes in fluid stagnation enthalpy. The energy transfer as work is therefore numerically equal to the change in stagnation enthalpy of the fluid between the inlet and the outlet of the turbomachine.

In a power-generating turbomachine, \( w \) is positive as defined so that \( \Delta h_o \) is negative, i.e., the stagnation enthalpy at the exit of the machine is less than that at the inlet. The machine puts out work at the shaft. In a power-absorbing turbomachine, \( \Delta h_o \) is positive. The stagnation enthalpy at the outlet will be greater than that at the inlet and work is done on the flowing fluid due to the rotation of the shaft. As already shown in the example preceding Eq. (1.7), in turbomachines handling compressible fluids, changes in static head cause negligible changes in total enthalpy. For these machines therefore, it is sufficiently accurate to write:

\[
w = -\Delta h_o = -\Delta(h + V^2/2). \quad \ldots (1.11c)
\]

In machines dealing with liquids, since the effects of changes in potential energy are large and changes in internal energy are negligible (as shown later, Example 1.3), the density is constant and one can write:

\[
w = -\Delta h_o = -\Delta(p/\rho + V^2/2 + gz). \quad \ldots (1.11d)
\]

In a turbomachine, the energy transfer between the fluid and the blades can occur only by dynamic action, i.e., through an exchange of momentum between the rotating blades (Fig. 1.4, location 3) and the flowing fluid. It thus follows that all the work is done when the fluid flows over the rotor-blades and not when it flows over the stator-blades. As an example, considering a turbomachine with a single stator-rotor combination shown schematically in Fig. 1.4, let points 1 and 2 represent respectively the inlet and the exit of the stator. Similarly, points 3 and 4 represent the corresponding positions for the rotor blades. Then ideally for flow between points 1 and 2, there should be no stagnation enthalpy changes since no energy transfer as heat or work occurs in the stator. Thus, \( h_{o1} = h_{o2} \). For flow between points 3 and 4 however, the stagnation enthalpy change may be negative or positive, depending upon whether the machine is power-generating or power-absorbing. Hence, \( h_{o3} > h_{o4} \) if the machines develops power and if \( h_{o3} < h_{o4} \), the machine needs a driver and absorbs power.
A large machine is generally a combination of stator-rotor stages of the type described above. If the effects of friction and other losses are neglected, there can be no stagnation enthalpy changes in any stator-blade or nozzle. The stator is thus essentially a flow-directing device in which only static enthalpy, kinetic and potential energies can change, leaving the stagnation properties unaltered. In the rotor stages, dynamic interaction occurs between the fluid and the blades leading to energy exchange as work and consequently, changes in stagnation properties.

If the system is perfectly reversible and adiabatic with no energy transfer as work, no changes can occur in the stagnation properties (enthalpy, pressure and temperature) between the inlet and the outlet of the machine. All turbomachines exchange work with the fluid and also suffer from frictional as well as other losses. The effect of the losses in a power-generating machine is to reduce the stagnation pressure and to increase entropy so that the net work output is less than that in an ideal process. The corresponding work input is higher in a power-absorbing machine as compared with that in an ideal process. In order to understand how this happens, consider the Second Law equation of state, $T ds_o = dh_o - v_o dp_o$. (This is the form of the equation $T ds = dh - v dp$, when applied to stagnation properties.) Also, $dh_o = -\delta w$, as demonstrated earlier (Eq. 1.11b). Hence,

$$-\delta w = v_o dp_o + T_o ds_o.$$  \hspace{1cm} (1.11e)

In a power-generating machine, $dp_o$ is negative since the flowing fluid undergoes a pressure drop when mechanical energy output is obtained. However, the Clausius inequality [15] requires that $T_o ds_o \geq \delta q$, and as $\delta q \approx 0$, in a turbomachine $T_o ds_o \geq 0$. The sign of equality applies only to a reversible process which has a work output $\delta w = -v_o dp_o > 0$. In a real machine, $T_o ds_o > 0$, so that $\delta w_i - \delta w = T_o ds_o > 0$ and represents the decrease in work output due to the irreversibilities in the machine. The reversible power-generating machine therefore exhibits the highest mechanical output of all the machines undergoing a given stagnation pressure change. A similar argument may be used to prove that if the device absorbs power, the work input needed to obtain a specified stagnation pressure rise is a minimum when the device is reversible. In this case, $\delta w_i - \delta w = T_o ds_o > 0$, so that both the ideal work $\delta w_i$ and the actual work $\delta w$ are negative, the actual work being larger than the ideal work in magnitude.
Example 1.2. A turbomachine handling liquid water is located 8 m above the sump level and delivers the liquid to a tank located 15 m above the pump. The water velocities in the inlet and the outlet pipes are respectively 2 m·s$^{-1}$ and 4 m·s$^{-1}$. Find the power required to drive the pump if it delivers 100 kg·min$^{-1}$ of water.

**Data:** Pump handling liquid water of constant density $\rho = 1000$ kg·m$^{-3}$, $z_1 = 8$ m, $z_2 = 15$ m, $V_1 = 2$ m·s$^{-1}$, $V_2 = 4$ m·s$^{-1}$, mass flow rate = 100 kg·min$^{-1}$.

**Find:** Power needed to drive the pump, $P$.

**Solution:** Liquid water may be considered as incompressible in the pressure range of operation of most pumps so that we can assume that the enthalpy is a function of density alone and is independent of temperature. With these assumptions, we get:

$$w = q - \Delta h_o = - \frac{\Delta p_o}{\rho},$$

$$= - \left[ \frac{p_2 - p_1}{\rho} + \frac{(V_2^2 - V_1^2)}{2} \right] + g(z_2 - z_1),$$

$$= \left[ 0 + \frac{(4^2 - 2^2)}{2} + 9.81(15 + 8) \right] = 231.6 \text{ J·kg}^{-1}.$$

Note the following:

(i) The pressure difference $(p_2 - p_1)$ is taken as zero, since both the sump from which the water is drawn and the delivery tank are open to atmosphere.

(ii) The value of $z_1$ is $-8$ m, since the measurements are made with respect to the location of the pump.

Ideal (minimum) power needed to drive the pump:

$$P = \dot{m}w = \frac{100}{60}(231.6) = 386 \text{ W}.$$

![Schematic diagram of pump](image_url)

The actual power needed to drive the pump will be larger than that calculated above due to losses in friction in the pipes, entry and exit losses, leakage, etc. The method of allowing for...
these losses and estimating more accurately the power needed to drive the pump (or for a turbine, power obtainable from it) will be considered in the following sections.

### 1.5 EFFICIENCY OF TURBOMACHINES

It has been seen above that the performance of a real machine is inferior to that of a frictionless and loss-free ideal machine. A measure of its performance is the efficiency, defined differently for power-generating and power-absorbing machines as given by the following equations:

\[
\eta_{pg} = \frac{\text{Actual Shaft Work Output}}{\text{Ideal Work Output}} = \frac{w_s}{w_i}, \quad \ldots(1.12)
\]

and,

\[
\eta_{pa} = \frac{\text{Ideal Work Input}}{\text{Actual Shaft Work Input}} = \frac{w_p}{w_s}, \quad \ldots(1.13)
\]

In the expressions above, \(w_s\) represents the shaft power output for the power-generating machine and the shaft-power input for the power-absorbing machine. For a loss-free system, the term \(w_i\), represents the ideal output if the machine generates power and the ideal power input if it absorbs power. The ideal work is calculated on the basis of isentropic processes throughout the system.

Generally speaking, losses occur in turbomachines due to: (a) bearing friction, windage, etc., all of which may be classified as mechanical losses, (b) unsteady flow, friction between the blade and the fluid, etc., which are internal to the system and may be classified as fluid-rotor losses. (There are other losses like leakage across blades, labyrinth leakage, etc. in addition to the above losses. These are covered under fluid-rotor losses.) If the mechanical and fluid-rotor losses are separated, the efficiencies written earlier may be rewritten in the following forms:

\[
\eta_{pg} = \frac{\text{Fluid-Rotor Work}}{\text{Shaft Work Output}} \times \frac{\text{Shaft Work Output}}{\text{Ideal Work Output}} = \frac{w_r}{w_s} \times \frac{w_s}{w_i}, \quad \ldots(1.14)
\]

and similarly,

\[
\eta_{pa} = \frac{\text{Ideal Work Input}}{\text{Rotor-Fluid Work}} \times \frac{\text{Rotor-Fluid Work}}{\text{Shaft-Work Input}} = \frac{w_p}{w_r} \times \frac{w_r}{w_s}, \quad \ldots(1.15)
\]

In Eq. (1.14), \(w_r\) is the energy transfer between the fluid and the rotor and would be the output at the shaft, if there were no mechanical losses due to windage, friction at the bearings etc. The quantity \(w_i/w_r\) is called the adiabatic, isentropic or hydraulic efficiency of the power-generating system, since \(w_r\) is always calculated on the basis of a loss-free isentropic flow. Hence, adiabatic efficiency for a power-generating machine may be rewritten as:

\[
\eta_a = \frac{\text{Mechanical Energy Supplied by the Rotor}}{\text{Hydrodynamic Energy Available from the Fluid}} \quad \ldots(1.16a)
\]
15

Similarly, for a power-absorbing machine, (Eq. (1.15) with adiabatic efficiency \( \eta = w_i/w_r \), it is seen that:

\[
\eta_p = \frac{\text{Hydrodynamic Energy Supplied to the Fluid}}{\text{Mechanical Energy Supplied to the Rotor}}. \quad \text{(1.16b)}
\]

The difference between \( w_s \) and \( w_r \) is expressed in terms of mechanical efficiency, defined by the equations:

\[
\eta_m = \frac{w_s}{w_r} = \frac{\text{Shaft Work Output}}{\text{Fluid-Rotor Work}} \quad \text{(from Eq. (1.14), \( pg \) machine).} \quad \text{(1.17a)}
\]

\[
\eta_m = \frac{w_s}{w_r} = \frac{\text{Rotor-Fluid Work}}{\text{Actual Work Input to Shaft}} \quad \text{(from Eq. (1.15), \( pa \) machine).} \quad \text{(1.17b)}
\]

Then,

\[
\eta_{pg} = \eta_m \quad \text{(1.18)}
\]

Usually, mechanical losses in large turbomachines do not exceed 1%. In very large machines dealing with hundreds of megawatts of power, (the kind used in large power stations), these losses may be smaller than 0.5%. Even in rather small machines with diameters of the order of 300 mm, mechanical efficiencies are 90-95% (unlike in reciprocating engines where mechanical efficiencies rarely exceed 83-85%). Moreover, mechanical losses are not strong functions of load and fluid states, since most turbines are governed to run at a constant speed. Hence, it is usual to assume the mechanical efficiency to be unity in many cases including those for power absorbing machines, unless it is stated otherwise. With this assumption, the overall efficiency (of all large turbomachines, power-generating or power-absorbing), equals its adiabatic efficiency, i.e.,

\[
\eta_{pg} = \eta_m \quad \text{and} \quad \eta_{pa} = \eta_r. \quad \text{(1.19)}
\]

In order to determine the adiabatic efficiency of a turbomachine during a test, it is necessary to specify the ideal work input or output by using the fluid states at the inlet and the outlet respectively. However, as is obvious, the ideal work may be calculated by using either the static or the stagnation properties of the fluid or, even by a combination of suitably chosen pairs of both. The ideal work based upon static states only or stagnation states only to specify the properties at the inlet and exit will not differ much if the inlet and exit fluid kinetic energies are not large. This used to be the case when steam turbines were first developed, so it sufficed then to evaluate all efficiencies based on static property changes. Indeed, calculations of Rankine cycle efficiencies [16] are based only on static enthalpy changes since the velocities of fluid flow at the entry and the exit of reciprocating engines are of negligible importance. In modern turbines however, the fluid at the inlet and the exit can be at high velocities and efficiencies based on stagnation properties may be of greater value in judging the performance of these devices. Referring to the \( h-s \) diagrams Fig. 1.5(a) for power-generating and Fig. 1.5(b) for power-absorbing turbomachines, the fluid has initially the static pressure and temperature determined by state 1, with state o1, as the corresponding stagnation state. After passing through the turbomachine, the final static properties of the fluid are determined by state 2, with o2 as the corresponding stagnation state. If the process were reversible, the final fluid static state would be 2', and the stagnation state would be o2'. The dashed-lines 1–2 in static coordinates and
o1–o2 in stagnation coordinates represent the real process in each of the two figures. The actual work input or output \( w_t \) is the quantity \( h_{o1} - h_{o2} \) whereas the ideal work \( w_i \), can be calculated by any one of the following four equations:

\[ (i) \quad w_{t-t} = h'_{o2} - h_{o1}, \quad \text{(initial and final states both total)}, \quad \ldots(1.20) \]

\[ (ii) \quad w_{t-s} = h'_2 - h_{o1}, \quad \text{(initial state total, final state static)}, \quad \ldots(1.21) \]

\[ (iii) \quad w_{s-t} = h'_{o2} - h_1, \quad \text{(initial state static, final state total)}, \quad \ldots(1.22) \]

\[ (iv) \quad w_{s-s} = h'_2 - h_1, \quad \text{(initial and final states static)}. \quad \ldots(1.23) \]

The proper equation for use is to be decided by the conditions of the turbomachine in question. For example, in a turbine, if the kinetic energy of the fluid can be used for the production of mechanical energy somewhere else and the kinetic energy at the inlet is negligible, one can use the static-to-total definition, (Eqs. 1.22, 1.24c), or the total-to-total definition (Eqs. 1.20, 1.24a). The results obtained from both the definitions will be nearly the same and will take account of all the useful energy for the evaluation of efficiency. However, if the exit kinetic energy is wasted, the appropriate measure will be static-to-static (Eq. 1.23, 1.24d), to increase the measure of ideal work and show the losses to the system. The Rankine cycle [16] still uses this measure since the steam emerging from the turbine is condensed and its kinetic energy is completely wasted.

Based on the calculations of mechanical work presented above, the following efficiencies for power-generating machines may be defined:

\[ (i) \quad \eta_{l-t} = \frac{h_{o1} - h_{o2}}{h_{o1} - h_{o2}'} \quad \ldots(1.24a) \]

\[ (ii) \quad \eta_{l-s} = \frac{h_{o1} - h_{o2}}{h_{o1} - h'_2} \quad \ldots(1.24b) \]

\[ (iii) \quad \eta_{s-t} = \frac{h_{o1} - h_{o2}}{h_1 - h_{o2}'} \quad \ldots(1.24c) \]

\[ (iv) \quad \eta_{s-s} = \frac{h_{o1} - h_{o2}}{h_1 - h'_2} \quad \ldots(1.24d) \]
All the above definitions are applicable to power-generating machines, since the actual work is in the numerator and the denominator contains the ideal work, based on the conditions of the system for which efficiency is being defined.

For power-absorbing machines, the applicable definitions of efficiency are the following:

(i) $\eta_{t-t} = \frac{(h_{o2}' - h_{o1})}{(h_{o2} - h_{o1})}$  \hspace{1cm} \ldots (1.25a)

(ii) $\eta_{t-s} = \frac{(h_2' - h_1)}{(h_2 - h_1)}$. \hspace{1cm} \ldots (1.25b)

(iii) $\eta_{s-t} = \frac{(h_{o2}' - h_1)}{(h_{o2} - h_{o1})}$. \hspace{1cm} \ldots (1.25c)

(iv) $\eta_{s-s} = \frac{(h_2' - h_1)}{(h_{o2} - h_{o1})}$. \hspace{1cm} \ldots (1.25d)

Example 1.3. For a power-absorbing turbomachine handling water, the total-total efficiency ($\eta_{t,t}$) is 0.70. During flow through the machine, the stagnation pressure of the water rises by 3.5 atm. Find for this machine, the actual mechanical input needed, the ideal energy input (total-to-total), the rise in temperature of water due to irreversibilities and the actual power input needed for a water-flow of 0.195 m$^3$.min$^{-1}$. Assume the mechanical efficiency to be 0.9.

Data: Pump with total-to-total efficiency $\eta_{t,t} = 0.70$, stagnation pressure rise $\Delta p_o = 3.5$ atm, fluid density $\rho = 1000$ kg.m$^{-3}$, mass flow rate $\dot{m} = \rho \dot{Q} = (1000)(0.195/60) = 3.25$ kg.s$^{-1}$, mechanical efficiency $\eta_m = 0.9$.

Find: Power input $P$.

Solution: Since the fluid is incompressible, $v_o = \text{const} = 1/\rho$.

For an isentropic compression, $(v_o \text{kg.m}^{-3}, \Delta p_o \text{ Pa})$:

$\Delta h_o' = v_o \Delta p_o = (1/1000)[(3.5)(1.0131)(10^5)] = 355 \text{ J.kg}^{-1}$.

From Eq. (1.25a), $\eta_{t,t} = \Delta h_o'/(h_{o2} - h_{o1}) = (355)/(h_{o2} - h_{o1}) = 0.7$.

Hydrodynamic energy at the rotor $= h_{o2} - h_{o1} = \Delta h_o'/\eta_{t,t} = 355/0.7 = 506.6 \text{ J.kg}^{-1}$.

Total power input: $P = \dot{m}w/\eta_m = (3.25)(506.6)/0.9 = 1825 \text{ W} = 1.829 \text{ kW}$.

If the rise in pressure during flow through the turbomachine had occurred isentropically, there would be no temperature change in the fluid since it is incompressible. A small part of the mechanical energy is dissipated into its thermal form due to irreversibilities and thus a temperature rise occurs. The temperature change is computed by using the equation:

$T_o ds_o = du_o + p_o dv_o = du_o = dh_o - v_o dp_o = 506.6 - 355 = 151.6 \text{ J.kg}^{-1}$.

(In the expressions above, the term $p_o dv_o$ is zero, since the density is constant and the specific volume cannot change either). Hence, with $c_v = 4187.2 \text{ J.kg}^{-1}\text{K}^{-1}$ for water, we obtain,

$du_o = c_v dT_o = 152 \text{ kJ.kg}^{-1}$ or, $dT_o = 151.6/4187.2 = 0.036^\circ \text{C}$.

This change is quite small since the liquid is incompressible and its specific heat is large. It is difficult to measure such temperature changes in liquids flowing through turbomachines. Gases on the other hand, experience large temperature changes (which can be readily measured), during flow through turbomachines.
Example 1.4. Steam enters a turbine at a static pressure, a static temperature and a flow velocity of 45 bar, 550°C and 200 m·s⁻¹ respectively. At the turbine exit, the static pressure, static temperature and velocity are 1 bar, 110°C and 250 m·s⁻¹ respectively. Neglecting heat transfer during the expansion process, calculate: (a) the total-to-total efficiency, (b) total-to-static efficiency and, (c) the static-to-static efficiency.

Data: Inlet static pressure \(p_1 = 45\) bar, static temperature \(T_1 = 550°C\), velocity \(V_1 = 200\) m·s⁻¹, exit static pressure \(p_2 = 1\) bar, static temperature \(T_2 = 110° C\) and, velocity \(V_2 = 250\) m·s⁻¹.

Find: (a) total-to-total efficiency \(\eta_{t-t}\), (b) total-to-static efficiency \(\eta_{t-s}\), and (c) static-to-static efficiency.

Solution: (a) By using superheated steam tables (Table A.3), we obtain at the initial static state 1, pressure 45 bar and temperature 550°C, a static enthalpy \(h_1 = 3555\) kJ·kg⁻¹ and an entropy \(s_1 = 7.173\) kJ·kg⁻¹·K⁻¹. Hence, total enthalpy (enthalpies in kJ·kg⁻¹):

\[
ho_1 = h_1 + V_1^2/2000 = 3555 + 200^2/2000 = 3575\text{ kJ·kg}^{-1}.
\]

At the exit, the properties are: \(p_2 = 1\) bar, \(T_2 = 110°C\) and velocity \(V_2 = 250\) m·s⁻¹.

The corresponding enthalpy \((h_2)\) at these conditions obtained from the superheated steam table A.3 through interpolation between tabulated values, is \(h_2 = 2696.5\) kJ·kg⁻¹. So,

\[
ho_2 = h_2 + V_2^2/2 = 2696.5 + 250^2/2000 = 2727.75\text{ kJ·kg}^{-1}\text{ and,}
\]

\[
w = ho_1 - ho_2 = 3575 - 2727.75 = 847.25\text{ kJ·kg}^{-1}.
\]

In order to determine the total-to-total efficiency, one should know \(ho_{2}^\prime\), the total enthalpy at the turbine exit when the expansion occurs isentropically from the initial state with \(p_1 = 45\) bar, \(T_1 = 550°C\), to a final state with \(p_2 = 1\) bar. It is usual to design the turbine to expand the steam so that it is slightly wet at the exit. Since the given exit temperature is only 10°C above the saturation temperature at the exit pressure of 1 bar, it is simplest to assume that the ideal expansion is likely to make it slightly wet and be below the saturation line. Let the steam quality at this state be denoted by \(x'\). The saturation properties of steam at the exit pressure are:

\[
h_f = 417.4\text{ kJ·kg}^{-1}, h_g = 2675\text{ kJ·kg}^{-1}, s_f = 1.3026\text{ kJ·kg}^{-1}\text{·K}^{-1}\text{ and, } s_g = 7.359\text{ kJ·kg}^{-1}\text{·K}^{-1}.
\]

We calculate the quality of steam by noting that the entropy remains constant during an isentropic expansion from state 1 to the final ideal state 2'. Therefore,

\[
x' = (s_1 - s_f)/(s_g - s_f) = (7.173 - 1.3026)/(7.359 - 1.3026) = 0.969.
\]

Since \(x' < 1\), the steam would be wet in an ideal expansion from state 1 to the pressure \(p_2 = 1\) bar. Then, for the static and stagnation enthalpies at state 2', we obtain:

\[
h_2' = h_f + x'(h_g - h_f) = 417.4 + 0.969(2675 - 417.4) = 2605\text{ kJ·kg}^{-1}.
\]

\[
ho_{2}' = h_2' + V_2^2/2 = 2605 + 250^2/2 = 2636.25\text{ kJ·kg}^{-1}.
\]

\[
h_{o1} - ho_{2}' = 3575 - 2636.25 = 1038.8\text{ kJ·kg}^{-1}.
\]

\[
\eta_{t-s} = w/(ho_{o1} - ho_{o2}') = 847.25/1038.8 = 0.816\text{ or } 81.6%.
\]
(b) To compute the total-to-static efficiency, one should compute $h_{o1} - h_{2}'$, where the point $2'$ represents the final ideal static state after an isentropic expansion from the state with $h_{o1} = 3575$, to a final pressure $p_2 = 1$ bar. This is found to be:

$$h_{o1} - h_{2}' = 3575 - 2605 = 975 \text{ kJ kg}^{-1}.$$

Hence,

$$\eta_{t-st} = \frac{(h_{o1} - h_{o2})}{(h_{o1} - h_{2}')} = \frac{847.25}{975} = 0.869 \text{ or } 86.9\%$$

It shows significant difference between efficiencies in (a) and (b).

This is an example of a situation where the total-to-total efficiency is considerably lower than the total-to-static efficiency, the difference being about 5.3%. It means that there is a large kinetic energy that is going to waste.

(c) For static-to-static efficiency, the isentropic expansion is from the initial static state $p_1 = 45$ bar, $T_1 = 550^\circ$C, to the final state $p_2 = 1$ bar.

$$h_1 - h_2' = 3555 - 2605 = 950 \text{ kJ kg}^{-1}.$$  

$$\eta_{s-st} = \frac{w}{(h_1 - h_2')} = \frac{847.25}{950} = 0.892 \text{ or } 89.2\%$$

Example 1.5. Air flows through an air turbine where its stagnation pressure is decreased in the ratio 5:1. The total-to-total efficiency is 0.8 and the air flow rate is 5 kg s$^{-1}$. If the total power output is 400 kW, find: (a) the inlet total temperature, (b) the actual exit total temperature, (c) the actual exit static temperature if the exit flow velocity is 100 m s$^{-1}$ and (d) the total-to-static efficiency $\eta_{t-st}$ of the device.

Data: Air as a perfect gas, inlet-to-exit total pressure ratio $p_{o1}/p_{o2} = 5$, total-to-total efficiency $\eta_{t-t} = 0.8$, $m = 5$ kg s$^{-1}$, $P = 400$ kW and $V_2 = 100$ m s$^{-1}$. 
Find: Stagnation temperatures (a) $T_o1$, (b) $T_o2$, (c) Exit static temperature $T_2$ and (d) Total-to-static efficiency, $\eta_{t-s}$.

Solution: (a) Work output/unit mass flow of air, $(c_p = 1.004 \text{ kJ.kg}^{-1}\text{K}^{-1})$.

$$w = -\Delta h_o = -c_p(T_o2 - T_o1) = P/\dot{m} = 400/5 = 80 \text{ kJ.kg}^{-1}$$

Thus, $T_o2 - T_o1 = -80.0/1.004 = -79.7 \text{ K}$.

However, since the stagnation pressure ratio is 5 and the total-to-total efficiency is 0.8,

$$\frac{T_o2'}{T_o1} = \left(\frac{p_{o2}}{p_{o1}}\right)^{(\gamma - 1)/\gamma} = (0.2)^{(0.4/1.4)} = 0.631.$$  

$$T_o1 - T_o2 = \eta_{t-s}(T_o1 - T_o2') = 0.8(1.0 - 0.631)T_o1 = 79.7.$$  

$$T_o1 = 270 \text{ K}.$$  

(b) $T_o2 = T_o1 - 79.7 = 270 - 79.7 = 190.3 \text{ K}$.

(c) For the static temperature, we have: $T = T_o - V^2/(2c_p)$, so that:

$$T_2 = T_o2 - V_o^2/(2c_p) = 190.3 - 100^2/[2(1004)] = 185.3 \text{ K}.$$  

(d) From parts (a) and (b) above,

$$T_o2' = 0.631T_o1 = (0.631)(270) = 170.4 \text{ K}.$$  

Hence, $T_o2' = T_o2' - V_o^2/(2c_p) = 170.4 - 100^2/[2(1004)] = 165.4 \text{ K}$.

$$\eta_{t-s} = (T_o1 - T_o2)/(T_o1 - T_o2') = (270 - 190.4)/(270 - 165.4) = 0.76.$$  

### 1.6 PERFORMANCE CHARACTERISTICS AND DIMENSIONAL ANALYSIS

One is interested in studying the performance characteristics of turbomachines due to variations in size, initial temperature, speed, etc., to determine their effects on output variables like volume per unit mass flow rate, power output per unit of input, efficiency, etc. In general, the performance depends upon several variables, some of the more important of which are listed in Table 1.1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dimension (Symbol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$ = A characteristic physical dimension, e.g., runner or rotor diameter.</td>
<td>Length, [L]</td>
</tr>
<tr>
<td>$N$ = Rotational Speed,</td>
<td>rad.s$^{-1}$, [T$^{-1}$]</td>
</tr>
<tr>
<td>$Q$ or $\dot{m}$ = Volumetric or mass-flow rate, vol./time or mass/time</td>
<td>[L$^3$.T$^{-1}$ or M.T$^{-1}$]</td>
</tr>
<tr>
<td>$E$ = $gH$, Energy per unit mass,</td>
<td>J.kg$^{-1}$, [L$^2$.T$^{-2}$]</td>
</tr>
<tr>
<td>$H$ = Head, Height of a column of fluid of given density. (Used in hydraulic machines)</td>
<td>Length, [L]</td>
</tr>
<tr>
<td>$P$ = Power input or output,</td>
<td>Watt, [M.L$^2$.T$^{-3}$]</td>
</tr>
<tr>
<td>$\rho$ = Fluid density,</td>
<td>Mass/volume, [M.L$^{-3}$]</td>
</tr>
<tr>
<td>$h$ = Fluid enthalpy per unit mass,</td>
<td>J.kg$^{-1}$, [L$^2$.T$^{-2}$]</td>
</tr>
<tr>
<td>$\mu$ = Dynamic viscosity of fluid,</td>
<td>[M.L$^{-3}$.T$^{-1}$]</td>
</tr>
</tbody>
</table>
In this connection, we limit our attention to geometrically and kinematically similar machines, so that we can perform a dimensional analysis to determine the least number of non-dimensional quantities that can be used to specify the performance of all machines of a specified type. Geometrically similar machines are those which are all of the same shape, but with dimensions every one of which may be determined by knowing those of a given machine and a scaling-ratio between the two. (As an example of similarity in two dimensions, one can consider two similar triangles with the same included angle between two set of corresponding sides. Here, the ratio between any two corresponding sides is the same as that between two other corresponding sides.) This means that due to the similarity in shapes, both are scale-models of each other and can be obtained by geometric and scaled contractions or expansions of others. This also implies that we can consider only one characteristic dimension of each machine, since knowledge of that dimension and the corresponding dimension of the model provides the scaling-ratio between the two and hence, that of every other dimension of interest. There is one further implication of geometric similarity: since all machine dimensions of relevance are supposed to scale, the clearance-to-diameter ratio as well as the roughness-to-diameter ratio should be the same in all the machines. No attempt is made in practice to maintain geometric similarity related to clearance and roughness and hence all the specifications of geometric similarity are not satisfied. This does not usually result in serious difficulties since the Reynolds numbers are large (> 10^6) so that the flow is highly turbulent and the differences in performance due to non-similarity of roughness and small clearances do not play a significant role in machine performance as a whole. It is only when the Reynolds number falls below 300,000 that deviations relating to non-conformity with all aspects of geometric similarity start to become significant.

A second requirement is that the velocity triangles of fluid flow through the machines be scaled versions of one another as well. For this to happen, the angles at the inlet and the exit of the stators and rotors of the two machines must be identical, as required by geometric similarity due to the similarity in shapes of all the components of the machines. In addition, the streamlines of flow and their distribution should be similar in the two machines. Such machines are said to be geometrically and kinematically similar. The simultaneous existence of geometric and kinematic similarity is specified by the term, dynamic similarity. Therefore, dynamic similarity implies that the machines under consideration are geometrically similar (they are scale-models of each other) and kinematically similar (at all points of interest, the velocity triangles and streamlines in the machines are also similar). Dynamic similarity is the critical requirement for the following observations and inferences to be valid.

Now, on considering any one of the dependent variables, \( G \), as a function of the control variables, the rotor diameter and the fluid properties, one can write:

\[
G = f(m, N, \rho, p, h, D, \mu, \ldots) \quad \ldots(1.26)
\]

Here, the term \( G \) stands usually for any one of the following four quantities:

(i) \( E \), the energy transfer per unit mass flow, equal to \( gH \) \((H = \) the height of a static column of fluid, referred to as the ‘head’ which represents the height of the reservoir from which the fluid is drawn before it enters a hydraulic turbine. If the device is a pump, the head may
be the height to which the incompressible fluid is lifted). If the fluid is compressible, the quantity $E$, represents the static or stagnation enthalpy per unit mass flow.

(ii) The power output or input $P$, depending upon whether the machine is a turbine or compressor/pump, and

(iii) The efficiency, $\eta$.

(iv) Certain other quantities like torque may also be included in the list. The fluid density, $\rho$, may be omitted if it can be readily calculated through the equation of state (from knowledge of the pressure and the temperature of the fluid). In the event it is a liquid, the density is invariant and depends only on the specified fluid. Then the volume flow rate can be used as an alternative to the mass-flow rate, since the product of density and volume is the mass. In order to simplify the discussions, from here onwards we assume the flow to be incompressible though later, while dealing with compressors the variables and parameters pertaining to compressible flow will be specified.

In order to reduce the number of variables involved in the functional representation, one carries out a dimensional analysis using the variables in Eq. (1.26), noting that there are only three independent dimensions, length, mass and time. If we confine attention to those listed in Table 1.1, there is a total of nine variables, of which we can ignore $H$, since $E$ is the same as $H$, except for the multiplier $g$ which is the standard acceleration due to gravity, a constant. Hence, there are eight variables and only three independent dimensions. For this case, Buckingham’s $\pi$ theorem asserts that there exist only five independent dimensionless $\pi$-groups, as given below:

A. $\pi_1 = E/(N^2D^2) = gH/(N^2D^2)$ —— The Head Coefficient. …(1.27)

The Head-coefficient, $\pi_1$, is a measure of the ratio of the fluid potential energy (column height $H$) and the fluid kinetic energy while moving at the speed, $u$ (rotational speed of the wheel). The term can be interpreted by noting that:

$$\pi_1 = \frac{gH}{ND^2} \propto \frac{mV^2}{mu^2}$$

= (Representative) $ke$ of fluid /$ke$ of fluid moving at rotor-tip speed.

This is a constant in dynamically similar machines. For a machine of specified diameter, the head varies directly as the square of the speed. (This is the second of the fan laws stated in Chap. 6, except that the fluid density is considered as a variable in fans.) In fans and pumps, the head coefficient represents the ratio of the actual head to the theoretical head at shut-off.

B. $\pi_2 = Q/(ND^3) = (Q/D^3)/(ND) = V/u$ …Flow (or Discharge) Coefficient …(1.28)

The Flow-coefficient $\pi_2$ represents the ratio of the representative fluid velocity $V$, to the wheel-tip speed, $u$, (since $D$, is the characteristic dimension and $V$ is a representative velocity of flow). It is also called the specific capacity and signifies the volumetric flow rate of the fluid through a turbomachine with a unit diameter runner, operating at unit-speed. The specific-capacity is constant for dynamically similar conditions. Hence, for a fan or pump of a certain diameter running at various speeds, the discharge is proportional to the speed. This statement is called the First fan law (Chapter 6). The reciprocal of $V/u$, denoted by the Greek symbol, $\phi$, is referred to, as the speed-ratio:
\[ \varphi = \frac{u}{V} = \frac{u}{(2gh)^{1/2}}, \quad \text{i.e.,} \quad \varphi = C'(D/H^{1/2}), \]  

\[ \ldots (1.29) \]

where \( C' \) is a constant of proportionality for dynamically similar machines. The speed-ratio represents the ratio of the runner tangential speed to the theoretical spouting velocity of the jet (jet velocity) under the static head acting on the machine. If the value of \( \pi_2 \) is common to several machines, it means that they all have a common speed-ratio, \( \varphi \), and hence velocity triangles of the same shape (Chapter 2), i.e., the triangles are similar and have the same included angle between two similar sides.

If we limit our attention to incompressible fluids, it is clear that the density does not vary with temperature. As a consequence, the local fluid pressure can replace enthalpy. For an incompressible fluid therefore, we have effectively reduced the number of \( \pi \)-terms from five to two, (since the term \( \pi_3 \) represents the Reynolds number which has very little effect on the other parameters and the term \( \pi_5 \) represents the Mach number which is important only in compressible flow). The term \( \pi_4 \) is related to power and efficiency as will be seen shortly [Eq. (1.34)]. Hence, we should be able to use pressure as the variable in place of enthalpy and as a consequence, try to correlate the terms \( \pi_1 \) and \( \pi_2 \) through a relation of the type:

\[ \pi_1 = F(\pi_2). \]  

\[ \ldots (1.30) \]

If this conjecture is correct, it should be possible to plot, for example, the head-coefficient against the discharge-coefficient and obtain one curve which is common to all dynamically similar machines. In practice, if the head-coefficient, \( E/(ND)^2 \) or \( gH/(ND)^2 \) is plotted against the discharge-coefficient, the experimentally obtained data points do fall close to a single curve. The observed scatter, if any, is usually due to the so-called scale-effects (e.g., clearance, roughness, etc.) which are not geometrically similar. In addition, the Reynolds number which is represented by the third non-dimensional parameter \( \pi_1 \), does have a slight influence on machine performance, even though the flow is highly turbulent. (Friction factor is not quite constant. Thus, fluid viscosity and friction do play a role in determining machine behavior, though not very significantly). Hence, it is sufficiently accurate to treat \( \pi_1 \) as a function of \( \pi_2 \) alone, except when the fluid is very viscous or when cavitation and such other abnormal conditions that distort the streamlines of flow exist.

For dynamically similar systems, it is not necessary to conduct experiments on full-scale systems to determine their performance characteristics. It is sufficient to test small-scale models and use the observed performance to determine their performance characteristics at other conditions. If, for example, through a test on a model of a centrifugal pump, one has a plot of the head versus discharge characteristic at a certain speed \( N_1 \), as shown in Fig. 1.6, it is possible to predict its performance at any other specified speed, \( N_2 \). This is due to the non-dimensional \( \pi \)-terms which remain the same for all dynamically similar conditions. Hence, one can write: \( gH/(ND)^2 = \text{const} \), \( Q/(ND^3) = \text{const} \), \( P/(\rho N^3 D^5) = \text{const} \), …, and similar expressions for the ratios of \( \pi \)-terms which are also of importance in fluid machinery. As an example, consider a point, say \( A \), (Fig. 1.6) on the \( H-Q \) characteristic at speed \( N_1 \). To find the head and the discharge at a point \( B \) where the speed is \( N_2 \), it is seen that \( H_B = (N_2/N_1)^2 H_A \) and \( Q_B = (N_2/N_1) Q_A \), where \( H_A \) and \( Q_A \) are respectively the head and the discharge at the point \( A \). Since the speeds \( N_1 \) and \( N_2 \) are known along with the head and the discharge at the point \( A \), one can calculate both the head and the discharge at the point \( B \). Then, the point \( B \) can be plotted on the figure.
It is also clear from the equations for $H$ and $Q$ that both the points $A$ and $B$ lie on a parabola $H = (\text{Const.})Q^2$ passing through the origin. In a similar way, a number of points $B_1, B_2, B_3, \ldots$, corresponding to points $A_1, A_2, A_3, \ldots$, can be obtained and the $H$-$Q$ characteristic at speed $N_2$ drawn. The characteristic at any other speed, $N_3$ may also be drawn by repeating the process and using $N_3$ in place of $N_2$ in the calculations mentioned above.

**Fig. 1.6.** Pump performance characteristics.

C. \[ \pi_3 = \rho ND^2/\mu = \rho(ND)D/\mu = \rho uD/\mu \] — Reynolds Number (at wheel speed) \ldots (1.31)

Equation (1.31) represents the Reynolds number, since the quantity $ND^2$ is proportional to $DV$ for similar machines that have the same speed-ratio, $\varphi$. For fluids like water, air or steam, the term $\pi_3$ is on the order of $10^6$ or greater. Under these conditions, the flow is highly turbulent and the performance of the machine becomes nearly independent of the Reynolds numbers, since the friction factor too becomes nearly constant at high Reynolds numbers. Moreover, various other losses such as due to shock at the entry, impact, turbulence and leakage affect machine performance more than the change in Reynolds number does. Hence, attempts to correlate efficiencies and other variables with Reynolds numbers have been unsuccessful. It is therefore usual to neglect the dependence of the other non-dimensional terms on $\pi_3$. Even though theoretical predictions which neglect the dependence of performance on Reynolds number agree with experiments quite well, it does not mean that during the pumping of high viscosity fluids like heavy oils, we can neglect the effect of viscosity. The Reynolds number may become very low resulting in changed patterns of streamlines and velocity vectors. Then, the model similarity laws have to be corrected for Reynolds number dependency.

Machines of different sizes handling oils and other viscous fluids undergo efficiency changes under varying load conditions. For this reason, Moody [13] has suggested an equation to determine turbine efficiencies from experiments on a geometrically similar model. This equation which permits the calculation of efficiencies for varying model sizes (valid for heads $< 150$ m) is given below:
\[ \eta = 1 - (1 - \eta_m)(D_m/D)^{1/5}, \]  
...(1.32)  

where, \( \eta \) = Efficiency of prototype of diameter \( D \),  
\( \eta_m \) = Efficiency of model of diameter \( D_m \).  

For heads larger than 150 m, the efficiencies of model and prototype are related by the equation:

\[ \frac{1 - \eta}{1 - \eta_m} = \left( \frac{D_m}{D} \right)^{0.25} \left( \frac{H_m}{H} \right)^{0.1}. \]  
...(1.33a)  

However, the exponent of \( (H_m/H) \) varies from 0.04 to 0.1 for varying conditions. So, this equation is difficult to use in practice.

For pumps, Wislicenus [14] gives the equation:

\[ 0.95 - \eta = (0.95 - \eta_m)[\log(32.7Q)/\log(32.7Q_m)]^2, \]  
...(1.33b)  

where, \( Q \) and \( Q_m \) are discharges in m\(^3\).s\(^{-1}\) for prototype and model pumps, respectively. This equation is valid when both the pumps have the same clearance and roughness.

Since the power outputs for the prototype and model hydraulic turbines are \( P = \eta \rho QgH \) and \( P_m = \eta_m \rho Q_m g H_m \), one may write the power-ratio \( P/P_m \) as:

\[ \frac{P}{P_m} = \frac{\eta}{\eta_m} \times \frac{Q}{Q_m} \times \frac{H}{H_m}. \]  
...(1.34a)  

Use of Eq. (1.27) for the head-coefficient and Eq. (1.28) for the flow coefficient yields Eq. (1.34b) and (1.34c) respectively, as seen below;

\[ \frac{N}{N_m} = \frac{D_m}{D} \left( \frac{H}{H_m} \right)^{1/2} \]  
...(1.34b)  

\[ \frac{Q}{Q_m} = \frac{N}{N_m} \left( \frac{D}{D_m} \right)^3. \]  
...(1.34c)  

On eliminating \( N/N_m \) in Eq. (1.34c) by using Eq. (1.34b), one gets:

\[ \frac{Q}{Q_m} = \left( \frac{H}{H_m} \right)^{1/2} \times \left( \frac{D}{D_m} \right)^2. \]  
...(1.34d)  

Finally, on substituting for \( Q/Q_m \) in Eq. (1.34a) from Eq. (1.34c), we get:

\[ \frac{P}{P_m} = \frac{\eta}{\eta_m} \times \left( \frac{D}{D_m} \right)^2 \times \left( \frac{H}{H_m} \right)^{3/2}. \]  
...(1.34e)  

Equation (1.34e) provides a relation that enables one to calculate the power output-ratio, \( P/P_m \) from a knowledge of the geometric ratio \( D/D_m \), head-ratio \( H/H_m \) and efficiency-ratio \( \eta/\eta_m \).

D. \( \pi_4 = P/(\rho N^3 D^5) \) —— Power Coefficient (or Specific power)  
... (1.35a)
The quantity $\pi_4$, is a non-dimensionalised form of the power output/input of a turbomachine and is again a very important term in turbomachinery. By utilizing the terms $\pi_1$ and $\pi_2$, and eliminating both $N$ and $D$ on the right side of the equation, it is seen that:

$$\pi_4 / (\pi_1 \pi_2) = P / (\rho g Q H) = \eta,$$  

...(1.35b)

since the last ratio is the actual power output $P$, divided by $\rho g Q H$, which is the theoretical maximum power realizable in the system.

An interesting aspect of the Head-Discharge curves (Fig. 1.6) discussed earlier is that it is possible to draw other curves such as the $P-Q$ curve by using the definition of the $\pi_4$ term given here. Since $\pi_4$ is constant for dynamic similarity, power is proportional to the cube of the rotational speed and to the fifth power of the characteristic dimension, Eq. (1.35a). At any point on the given $H-Q$ curve at the speed $N_1$, the power input can be calculated by using Eqs. (1.35a), (1.35b). Hence, a curve of $P$ against $Q$ at the given speed $N_1$ can be drawn from the $H-Q$ characteristic and this may be used along with the knowledge that $Q \propto N$ and $P \propto N^5$, to obtain the $P-Q$ curve at any other speed. This curve should be regarded as approximate since the efficiency of the pump or turbine may not be constant at all speeds, so that $P$ is not exactly proportional to $\rho Q g H$. If the pump efficiency is also known as a function of speed and discharge, a more accurate estimate of the power-discharge characteristic may be obtained.

The last of the $\pi$-terms obtainable with the variable list given earlier in Table 1.1 is:

E. $\pi_5 = (gH/h)^{1/2}$ —— Mach Number  

...(1.36)

It is seen here that the term $\pi_5$ is the square root of the ratio of $E/h$, and it will now be shown that this is indeed a representation of Mach number for the flow of a perfect gas. To this end, we note that $E = gH$ has the units of the square of a velocity, the velocity being that of the fluid entering the turbine from a source at a head, $H$. In the denominator, $h = c_v T$, is the specific enthalpy of a perfect gas. This is clearly proportional to $\gamma RT$, where $\gamma$ is the ratio of specific heats and $R$, is the perfect gas constant. Hence, the denominator in the expression can be replaced by the square of the velocity of sound ‘$a$’ in a perfect gas and we have the Mach number, $M = V/a$. This term is of great importance in the study of compressible flow of a perfect gas. It is of no consequence in the study of pumps and turbines which use incompressible fluids like water.

It should be clear from what has been said that the constancy in the magnitude of the $\pi$-terms implies dynamic similarity, i.e., similarity in every one of the following: geometry, flow fields, force fields and velocity fields. It is therefore not sufficient just to have two geometrically similar machines and expect the non-dimensional $\pi$-terms to be equal. As stated earlier, non-similarity in flow fields can develop if the Reynolds numbers of the two fields are vastly different from each other so that the flow is laminar in one and turbulent in another. Evidently, in this case, dynamic similarity will not exist and the modeling laws of the type specified earlier cannot be expected to hold.

The Specific-Speed: A dimensionless term of extremely great importance in incompressible flow devices is obtained by manipulating the discharge and head-coefficients to eliminate the
characteristic dimension $D$. Assuming as usual that the speed of rotation $N$, is in RPM, (unit used in engineering practice), the dimensionless form of the specific-speed may be defined by any one of the following expressions:

$$\Omega = \pi^{1/2}/\pi_1^{3/4} = \text{(Flow Coefft.)}^{1/2}/\text{(Head-coefft.)}^{3/4} = N\sqrt{Q/(gH)^{3/4}}, \text{ or,} \quad \ldots(1.37a)$$

$$\Omega = \pi^{1/2}/\pi_1^{3/4} = (\pi N/30)\sqrt{Q/(gH)^{3/4}} = \omega\sqrt{Q/(gH)^{3/4}}. \quad \ldots(1.37b)$$

Equation (1.37a) represents the form in which the specific-speed is expressed in very many older texts and is not consistent with SI units. Equation (1.37b) however is consistent with SI units since the rotational speed $N$, in RPM has been converted to rad-s$^{-1}$.

If $Q$ is eliminated in the last part of Eqs. (1.37a) and (1.37b) by substituting $Q = P/(\eta \rho g H)$, the expressions for $\Omega$ assume the forms:

$$\Omega = N\sqrt{P/[(\eta \rho)^{1/2}(gH)^{5/4}]} \quad \text{and,} \quad \ldots(1.38a)$$

$$\Omega = (\pi N/30)\sqrt{P/[(\eta \rho)^{1/2}(gH)^{5/4}]} = \omega\sqrt{P/[(\eta \rho)^{1/2}(gH)^{5/4}]], \quad \ldots(1.38b)$$

Both of the above sets of expressions may be treated as definitions of $\Omega^4$, though commonly, Eq. (1.37a) is used with respect to pumps and Eq. (1.38a), for turbines. In all the equations above, $N$ should be expressed in RPM$^5$.

The parameter $\Omega$ has the advantage of being independent of diameter. It relates a combination of $N$, $Q$ and $H$ or, $N$, $P$ and $H$ for all conditions where dynamic similarity exists in machines of similar shapes.

The non-dimensional parameter $\Omega$, though of extremely significant, is not commonly used. This is mostly due to the long-standing practice of writing the expressions for specific-speed in dimensional forms by noting that $g$, the acceleration due to gravity, is a constant independent of the fluid and machine characteristics and the density, $\rho$, is a constant for a given incompressible fluid. If these two quantities are discarded from the expressions for $\Omega$, and the rotational speed is expressed in RPM instead of rad-s$^{-1}$, we get two other quantities, $n_s$ and $N_s$, both with dimensions and referred to as specific-speeds in engineering literature. These are defined by the following equations:

$$n_s = \text{Pump Specific-Speed} = N\sqrt{Q/H^{3/4}} \quad \text{and,} \quad \ldots(1.39)$$

$$N_s = \text{Turbine Specific-Speed} = N\sqrt{P/H^{5/4}}. \quad \ldots(1.40)$$

In the expression for $n_s$ in SI, Metric and US units, $N$ represents the rotational speed in RPM (revolutions per minute). In SI units as usual, the flow through the pump is $Q$ (m$^3$.s$^{-1}$),

$^4$ In SI units (Systeme Internationale de unites), the rotational speed corresponding to $N$ RPM becomes $2\pi N/60$ or $\pi N/30$ rad-s$^{-1}$. (RPM is not a recognized SI unit). Hence wherever RPM appears, it should be replaced by $\pi N/30$ rad-s$^{-1}$. However, in order to maintain the common terminology in use for a long time, authors of text-books generally express rotational speeds in RPM. For this reason, in this book too we shall be sticking to the usual convention and express rotational speed, $N$, in RPM.

$^5$ Some authors express $N$ in revolutions per second to calculate the non-dimensional specific speed in Eqs. (1.37a) and (1.38a). This leads to extremely small magnitudes for $\Omega$. 
and the head is $H(m)$. The specific-speed of a pump may therefore be defined as the speed in RPM of a geometrically similar pump, discharging 1 m$^3$s$^{-1}$ of water against a head of 1 m. This definition can be used to derive the same expression as above for the pump specific-speed, $n_s$, by noting that:

$$Q \propto AV_f \propto D^2 H^{1/2}, \text{ and } u = \pi DN/60 \propto H^{1/2}. \text{ Hence, } D \propto H^{1/2}/N \text{ and,}$$

$$Q \propto (H/N^2) \times H^{1/2} = H^{3/2}/N^2, \text{ or, } Q = C_1 H^{3/2}/N^2, \quad \text{(1.41a)}$$

$C_1$ being a constant of proportionality to be determined.

In the paragraph above, it has been shown that specific-speed is the speed at which a geometrically and dynamically similar pump runs while working under a unit head (1 m) with a flow of (m$^3$s$^{-1}$). Hence, to evaluate $C_1$, we substitute $H = 1$ and $Q = 1$ and set $N = n_s$, in Eq. (1.41a) to obtain:

$$C_1 = n_s^2 \text{ and } n_s = N Q^{1/2}/H^{3/4}. \quad \text{(1.41b)}$$

In SI units, pump specific-speeds defined this way range between 20 and 300.

The second definition in the form, $N_s = N \sqrt{P/H^{5/4}}$, is used to calculate the specific-speeds of turbines. It is assumed that water is the fluid. Here again, $N_s$ is the speed in RPM, $P$, the power in kW and $H$, the head in metres. The specific-speed of a turbine may therefore be defined as the rotational speed of a geometrically similar turbine operating under a head of 1 m and producing 1 kW of power. It is to be noted that the term for efficiency is also absent in Eq. (1.39). Any parameter which varies with the operating point cannot be very useful when there are too many variables that affect it strongly. For the specific-speed to be of any practical value, it is therefore necessary to pick a specified operating point that denotes a characteristic efficiency – its maximum (or design value). This point is chosen since the design is always for operation at maximum efficiency and best performance.

As for pumps, where the volumetric flow rate was related to the head and specific-speed, we relate now the power developed to discharge and head as well as efficiency to write first the expression:

$$P = \eta \rho g Q H \propto (H^{3/2}/N^2)H = H^{5/2}/N^2, \text{ or, } P = C_2 \eta H^{5/2}/N^2, \quad \text{(1.41c)}$$

where $C_2$ is a constant of proportionality, to be determined by noting that the specific-speed is the speed at which a geometrically and dynamically similar turbine runs while operating under a head of 1 m and producing unit power (1 kW in SI). Then, on substituting $P = 1$, $H = 1$, and $N = N_s$, in Eq. (1.41c), we get $N_s^2 = C_2 \eta$. Since the efficiency varies with the point of operation, we assume further that the best performance is obtained under the unit head, unit output condition, i.e., the efficiency $\eta = \eta_{m'}$, at these conditions. With this provision, we obtain, for geometrically and dynamically similar turbines all of which have the same $\eta_{m'}$, $C_2' = C_2 \eta_{m'} = N_s^2$, and the expression for turbine specific-speed becomes:

$$N_s = N P^{1/2}/H^{5/4}. \quad \text{(1.41d)}$$

When this is done, it is found that each class of turbomachines has its maximum efficiency within a narrow range of specific-speeds which is different for different classes of machines. The specific-speeds for various classes of turbomachines are as indicated in Table 1.2. The table shows
that each type of machine works in a narrow range of specific-speeds and also has a maximum efficiency which can be independently specified. What is interesting is that this statement is true for compressible and incompressible flow machinery, though most often, the specific-speed is used as a parameter referring to incompressible flow devices. (In gas turbine design, the non-dimensional specific-speed is employed as the parameter with compressible flow running to $M \sim 1$.) As seen from the table, the range of specific-speeds of most machines is between 3 (for very high head Pelton wheels) and 900 (for low head propeller and Kaplan turbines).

Table 1.2. Ranges of Specific-speeds and Efficiencies of Turbomachines

<table>
<thead>
<tr>
<th>Turbomachine</th>
<th>$\Omega$</th>
<th>$n_s$ (52.9 $\Omega$)</th>
<th>$N_s$ (165.8 $\Omega$)</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pelton Wheel[8]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>– Single jet</td>
<td>0.02 – 0.19</td>
<td>4 – 32</td>
<td>87 – 88%</td>
<td></td>
</tr>
<tr>
<td>– Twin jet</td>
<td>0.1 – 0.3</td>
<td>16 – 50</td>
<td>86 – 88%</td>
<td></td>
</tr>
<tr>
<td>– Four jet</td>
<td>0.14 – 0.39</td>
<td>23 – 65</td>
<td>85%</td>
<td></td>
</tr>
<tr>
<td>Francis Turbine [8]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>– Radial flow</td>
<td>0.39 – 0.65</td>
<td>65 – 110</td>
<td>90 – 92%</td>
<td></td>
</tr>
<tr>
<td>– Mixed flow</td>
<td>0.65 – 1.2</td>
<td>110 – 200</td>
<td>93%</td>
<td></td>
</tr>
<tr>
<td>– Mixed flow</td>
<td>1.2 – 1.9</td>
<td>200 – 315</td>
<td>93 – 91%</td>
<td></td>
</tr>
<tr>
<td>– Mixed flow</td>
<td>1.9 – 2.3</td>
<td>315 – 385</td>
<td>91 – 89%</td>
<td></td>
</tr>
<tr>
<td>Propeller Turbine [9] (axial)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>– Radial flow</td>
<td>1.6 – 3.6</td>
<td>265 – 600</td>
<td>93 – 91%</td>
<td></td>
</tr>
<tr>
<td>Kaplan Turbine (axial)</td>
<td>2.7 – 5.4</td>
<td>450 – 900</td>
<td>94 – 87%</td>
<td></td>
</tr>
<tr>
<td>Centrifugal Pumps [4]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>– Turbine Pump (slow)</td>
<td>0.24 – 0.47</td>
<td>12 – 25</td>
<td>70 – 78%</td>
<td></td>
</tr>
<tr>
<td>– Volute Pump (Med.)</td>
<td>0.38 – 0.95</td>
<td>20 – 50</td>
<td>74 – 86%</td>
<td></td>
</tr>
<tr>
<td>– Volute Pump (Fast)</td>
<td>0.95 – 1.8</td>
<td>50 – 95</td>
<td>86 – 78%</td>
<td></td>
</tr>
<tr>
<td>Mixed Flow Pump [4]</td>
<td>1.80 – 4.0</td>
<td>95 – 210</td>
<td>79 – 76%</td>
<td></td>
</tr>
<tr>
<td>Radial Flow</td>
<td>0.4 – 0.62</td>
<td>21 – 32</td>
<td>68 – 80%</td>
<td></td>
</tr>
<tr>
<td>Compressors</td>
<td>0.62 – 1.4</td>
<td>32 – 74</td>
<td>80 – 54%</td>
<td></td>
</tr>
<tr>
<td>Axial Flow Turbines [4] (Steam and Gas)</td>
<td>0.35 – 1.9</td>
<td>18 – 315</td>
<td>83 – 88%</td>
<td></td>
</tr>
<tr>
<td>Axial Compressors and Blowers</td>
<td>1.4 – 2.3</td>
<td>74 – 120</td>
<td>72 – 88%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.3 – 20.</td>
<td>120 – 1050</td>
<td>88 – 72%</td>
<td></td>
</tr>
</tbody>
</table>

Though a given rotor of a specified geometric shape and fixed flow conditions has the same specific-speed no matter what its dimensions are, the specific-speed cannot, by itself, define the shape of the rotor. This is also evident and may be inferred from Table 1.2, where it is seen that there exist narrow specific-speed ranges where different types of rotors have their maximum
efficiencies. In spite of this, especially in incompressible flow handling machinery, it is possible to guess from the specific-speed, the approximate rotor shape. This statement is well illustrated by Figs. 1.7(a) and (b) which exhibit certain pump impeller shapes and their expected efficiencies at various flow rates. Further, small specific-speed rotors are rather narrow and have small openings whereas large specific-speed rotors have wide openings and are expected to have large flow rates. Thus, a centrifugal pump has a nearly pure radial outward flow. The area at the inlet is small. The flow rate is small because of the small inlet area but the head against which it works is high, because a relatively large rotor is needed to accommodate the flow \((H \propto D^2)\). The specific-speed is thus small. A volute or mixed-flow pump has a bigger opening because of its mixed-flow characteristic though the head developed is not as large as that of the turbine pump. Its specific-speed is higher than that of the turbine-pump. At the extreme end is the axial-flow pump, which has a relatively large flow area and therefore a considerable volume flow rate. The head it develops is therefore small compared with that of radial-flow pumps. Its specific-speed is very large.

![Fig. 1.7(a): Efficiency as a function of specific-speed in pumps. (After [17])](image)

![Fig. 1.7(b): Impeller shape variation with specific-speed in pumps. (After Troskolanski [17])](image)

Similarly, in Chapters 5 and 7, it will be seen that the specific-speed determines the approximate shapes of the rotors of turbines as well, in addition to their efficiencies, heads, etc. Consider for example the Pelton wheel which is a low specific-speed, high head turbine. The volumetric flow rate is small since the turbine utilizes one or more nozzles from which the fluid emerges as jets. The Francis turbine and the adjustable blade Deriaz together cover a wide range of
specific-speeds. Both are reaction turbines unlike the Pelton wheel and are suitable for intermediate heads. The axial-flow propeller and Kaplan turbines operate at low heads and need large fluid flow rates to produce reasonable amounts of power. Their specific-speeds are therefore high.

It is seen from the observations above that not only does the shape of the rotor determine the specific-speed, but also the specific-speed by itself fixes the approximate rotor shape as well as its efficiency. At the start of design, it may therefore be assumed that the specific-speed determines the rotor shape, at least in hydraulic machines. This assumption has to be checked as the design progresses further.

Since the practice is to define specific-speeds differently for pumps and for turbines [Eqs. (1.39) and (1.40)], their magnitudes differ according to the system of units. The units specified earlier (\(N\) in RPM, \(H\) in m, \(Q\) in m\(^3\).s\(^{-1}\) and \(P\) in kW) are for the SI system. In Metric units, the head is expressed in m, flow in m\(^3\).s\(^{-1}\) (or L/min), the power in Metric Horse Power (MHP = 0.736 kW), and the rotational speed in RPM. In USCU, the head is expressed in feet, the discharge in ft\(^3\)/s (or gallons/min), the power in HP (550 ft-lb/s), and the rotational speed in RPM. As such, the values of specific-speeds given in one set of units (say SI), are not the same as those given in another set of units, (say Metric or US). This fact should be borne in mind while reading the older books on turbomachines. To facilitate conversion from unit to unit, the relations among the specific-speeds for the three systems of units, SI, Metric and US, are given below:

**Pump Specific-Speeds:**

\[
2.438 \, n_s \text{ (SI)} = 2.438 \, n_s \text{ (Metric)} = n_s \text{ (US)}. \quad \text{(1.42a)}
\]

In these equations, the volumetric flow rate is expressed in m\(^3\).s\(^{-1}\), both in SI and Metric units, and in ft\(^3\)/s in US customary units. Very often however, the flow volume is expressed in US gallons/min. In that case, the pump specific-speeds in the three sets of units are related as shown below:

\[
51.65 \, n_s \text{ (SI)} = 51.65 \, n_s \text{ (Metric)} = n_s \text{ (US)}. \quad \text{(1.42b)}
\]

**Turbine Specific-Speeds:**

\[
N_s \text{ (SI)} = 0.8576 \, N_s \text{ (Metric)} = 3.813 \, N_s \text{ (US)}. \quad \text{(1.42c)}
\]

These specific-speeds assume that the fluid flowing through the machine is water of standard density. Finally, provided below are the relations between \(\Omega\) and the two specific-speeds, one of the pump and the second of the turbine in SI units:

\[
n_s \text{ (SI)} = 51.6 \, \Omega, \quad N_s \text{ (SI)} = 165.8 \, \Omega. \quad \text{(1.42d)}
\]

Here, \(n_s\) has the dimensions \([L^{3/4}T^{-3/2}]\) and \(N_s\), the dimensions \([M^{1/2}L^{-1/4}T^{-5/2}]\).

**Unit Quantities:** In hydraulic turbines, it is usual to define quantities referred to as unit flow, unit power, unit speed, etc., which are the values of the quantities under consideration per unit head. They are usually used in turbine design. The definitions of these quantities and the equations defining them are given below.

(i) **Unit Flow:** Unit flow is the flow that occurs through the turbine while working under unit head, the speed being that at design point. Since in all similar turbomachines, \(Q/(ND^3) = \text{Const.}\) and \(gH/(N^2D^2) = \text{const.}\), one has:

\[
[Q/(ND^3)]/[gH/(N^2D^2)]^{1/2} \quad \text{or} \quad Q/\sqrt{HD^2} = \text{const.}
\]
For a given turbine, D is a constant so that: \( \frac{Q}{\sqrt{H}} = \text{const.} \), which is referred to as *unit flow*. Hence,

**Unit Flow or Unit Discharge**, \( Q_1 = \frac{Q}{\sqrt{H}} = \text{const.} \) \hspace{1cm} \ldots (1.43a)

(ii) **Unit Power**: Unit Power is the power developed by (or supplied to) the hydraulic machine while working under a unit head, assuming operation at design speed and efficiency:

\[ P_1 = \frac{P}{H^{3/2}}. \] \hspace{1cm} \ldots (1.43b)

(iii) **Unit Speed**: Unit Speed is that speed at which the machine runs under unit head. Clearly, since \( gH/(N^2D^2) = \text{const.} \) for dynamically similar machines and the runner diameter is fixed,

\[ N_1 = \frac{N}{(\sqrt{H})} = \text{const.} \] \hspace{1cm} \ldots (1.43c)

Other unit quantities like unit force, and unit torque are defined at times. These are not commonly used in design.

**Example 1.6.** An axial-flow pump with a rotor diameter of 300 mm handles liquid water at the rate of 162 \( \text{m}^3\cdot\text{h}^{-1} \) while operating at 1500 RPM. The corresponding energy input is 125 J.kg\(^{-1}\), the total-to-total efficiency being 75%. If a second geometrically similar pump with a diameter of 200 mm operates at 3000 RPM, what are its: (a) flow rate, (b) change in total pressure and (c) input power?

**Data:** Pump rotor \( D_1 = 0.3 \text{ m} \), \( Q_1 = 162 \text{ m}^3\cdot\text{h}^{-1} \), \( N_1 = 1500 \text{ RPM} \). \( \eta_{t-t} = 0.75 \). Pump output \( w_1 = 125 \text{ J.kg}^{-1} \). Second pump, \( D_2 = 0.2 \text{ m} \), \( N_2 = 3000 \text{ RPM} \).

**Find:** For the second pump, (a) Volumetric flow rate \( Q_2 \), (b) the total pressure change, \( \Delta p_0 \) and (c) Power input, \( P_2 \).

**Solution:**

(a) For dynamic similarity between the two pumps,

\[ \pi_2 = Q_1/(N_1D_1^3) = Q_2/(N_2D_2^3), \]

i.e., \( Q_2 = Q_1 N_2 D_2^3/(N_1 D_1^3) = (162)(3000)(0.2)^3/[(1500)(0.3)^3] = 96 \text{ m}^3\cdot\text{h}^{-1} \).

(b) Since the head-coefficient is constant, Eq. (1.27) yields

\[ E_2 = E_1(N_2 D_2^2)/(N_1 D_1)^2 = (125)(3000 \times 0.2)^2/(1500 \times 0.3)^2 = 222 \text{ J.kg}^{-1}. \]

Change in total pressure:

\[ \Delta p_0 = \rho_1 E_2 = (1000)(0.75)(222) = 1.66 \times 10^5 \text{ N/m}^2 = 1.67 \text{ bar} \]

(c) Input power,

\[ P = \rho Q_2 E_2 = (1000)(96/3600)(0.222) = 5920 \text{ W} = 5.92 \text{ kW}. \]

**Example 1.7.** An axial-flow turbine handling air operates with a total pressure ratio of 3:1 and an inlet total temperature of 1000 K. The diameter and rotational speed of the turbine are 300 mm and 16,000 RPM, the total-to-total efficiency being 83%. Find the power output/(kg\cdot s\(^{-1}\)) of air flow, if the rotor diameter is reduced to 200 mm and the rotational speed to 12,000 RPM.

**Data:** Axial-flow turbine: \( p_{o1}/p_{o2} = 3 \), \( T_{o1} = 1000 \text{ K} \), \( D_1 = 0.3 \text{ m} \), \( N_1 = 16,000 \text{ RPM} \). \( \eta_{t-t} = 0.83 \). \( D_2 = 0.2 \text{ m} \), \( N_2 = 12,000 \text{ RPM} \).

**Find:** Work per unit mass, \( w \).

**Solution:** Since \( p_{o1}/p_{o2} = 3 \),

\[ T_{o2}' = T_{o1}(p_{o2}/p_{o1})^{(y-1)/y} = (1000)(3)^{-0.4/1.4} = 730.6 \text{ K} \]

\[ T_{o1} - T_{o2} = (T_{o1} - T_{o2}') \eta_{t-t} = (1000 - 730.6)(0.83) = 223.6 \text{ K}. \]
Example 1.8. The quantity of water available for a hydroelectric power station is 260 m³.s⁻¹ and a head of 1.73 m. If the speed of the turbines is to be 50 RPM and the efficiency 82.5%, find the number of turbines required. Assume \( N_s = 760 \). (Madras University, 1957).

**Data:** \( Q = 260 \text{ m}^3\cdot\text{s}^{-1}, H = 1.73 \text{ m}, N = 50 \text{ RPM}, \eta = 0.825, N_s = 760. \)

**Find:** Number of turbines required.

**Solution:** Since the specific-speed of the turbine is \( N_s = N \sqrt{P/H^{5/4}} \), the power
\[
P = N_s^2 H^{3/2}/N^2 = (760)^2(1.73)^{3/2}/50^2 = 909.5 \text{ kW/turbine}.
\]

Total Power = \( \eta \rho Q g H = (0.825)(1000)(260)(9.81)(1.73) = 3.64 \times 10^6 \text{ W} = 3640 \text{ kW} \).

Number of turbines needed = \( 3640/909.5 = 4. \)

Example 1.9. A small-scale model of a hydraulic turbine runs at a speed of 250 RPM under a head of 23 m and produces 8.25 kW as output. Assuming a total-to-total efficiency of 0.79 and that the model and turbine efficiencies are related by the Moody formula, find the power output of the actual turbine which is 6.5 times the size of the model. Specify the type of runner (Pelton, Francis or Kaplan), you would use in this case.

**Data:** \( N_m = 250 \text{ RPM}, H_m = 23 \text{ m}, P_m = 8.25 \text{ kW} \) and, \( \eta_m = 0.79, \frac{D}{D_m} = 6.5. \)

**Find:** Power output of the prototype, \( P \) and the type of runner, Pelton, Francis or Kaplan.

**Solution:** In scaling from the actual turbine to the model, it is usual to reduce the head in proportion to the size of the model. Hence, it may be assumed that the head on the turbine is:
\[
H = 23 \times 6.5 = 150 \text{ m}.
\]

Since the head on the prototype is 150 m, Moody’s first formula Eq. (1.32), can be used to compute the efficiency of the prototype. We write:
\[
\eta = 1 - (1 - \eta_m)\left(\frac{D}{D_m}\right)^{1/5} = 1 - (1 - 0.79)(1/6.5)^{1/5} = 0.856.
\]

Power output of the actual turbine (Eq. 1.34e):
\[
P = (\eta/\eta_m)\left(\frac{D}{D_m}\right)^2\left(\frac{H}{H_m}\right)^{3/2}P_m
= (0.856/0.79)(6.5)^2(6.5)^{3/2}(8.25) = 6258.9 \text{ kW}.
\]
(Note: It has been assumed here that similarity equations may be applied and the power incremented in proportion to the machine efficiency).

Expected speed of the turbine runner (Eq. 1.34b), with assumed \( H/H_m = D/D_m \) as above:
\[
N = N_m\left(\frac{D}{D_m}\right)(H/H_m)^{1/2} = 1320(1/6.5)6.5^{1/2} = 517.7 \text{ RPM}.
\]

Specific-Speed:
\[
N_s = N\sqrt{P/H^{5/4}} = 517.7(6258.9)^{1/2}/(150)^{5/4} = 78.5.
\]

As seen from Table 1.2, this specific-speed indicates that the runner is of the radial (Slow-speed Francis) type.

Example 1.10. A hydraulic turbine develops 20 MW under a head of 25 m while running at 60 RPM. The hydraulic efficiency is 0.9. Find its dimensionless specific-speed and its specific speed in SI units.

**Data:** Power \( P = 20 \text{ MW} = 2 \times 10^7 \text{ W}, \text{ head } H = 26 \text{ m}, \text{ density } \rho = 1000 \text{ kg.m}^{-3}, \)

\[
E_1 = w_1 = c_p\Delta T_o = (1004)(223.6) = 224,494 \text{ J.kg}^{-1}.
\]
\[
E_2 = E_1(N_2D_2)^2/(N_1D_1)^2 = (224,494)(12000 \times 0.2)^2/(16000 \times 0.3)^2
= 56,123 \text{ J.kg}^{-1} \text{ or, } w_2 = 56.123 \text{ kJ.kg}^{-1}.
\]
Hydraulic efficiency $\eta = 0.9$, RPM, $N = 60$.

**Find:** Dimensionless Specific-speed and its specific speed in SI units.

**Solution:** From Eq. (1.38b) one gets:

$$\Omega = (2\pi N/60)\sqrt{P/[(\eta\rho)^{1/2}(gH)^{5/4}]} = 2\pi[(2 \times 10^7/(0.9 \times 1000))]^{1/2}/(9.81 \times 20)^{5/4} = 1.276.$$ 

This is the dimensionless specific-speed with the rotational speed expressed in rad-s$^{-1}$.

(In SI units with rotational speed expressed in RPM, we have:

$$N_s = 165.8 \Omega = 165.8 \times 1.276 = 211.6$$

This specific-speed corresponds to a mixed-flow medium-speed Francis turbine.

**Nomenclature**

- $a =$ Speed of sound, m·s$^{-1}$
- $c_p =$ Specific heat at constant pressure, J·kg$^{-1}$K$^{-1}$
- $D =$ Characteristic dimension (usually diameter)
- $E =$ Energy per unit mass = $gH$, J·kg$^{-1}$
- $g =$ Standard acceleration due to gravity, 9.8066 m·s$^{-2}$
- $h =$ Specific enthalpy, J·kg$^{-1}$
- $H =$ Head, in m.
- $ke =$ Kinetic energy per unit mass, J·kg$^{-1}$
- $m =$ mass, kg; $\dot{m} =$ mass flow rate, kg·s$^{-1}$
- $M =$ Mach number, $V/a$
- $n_s =$ Compressor/Pump Specific-speed = $N\sqrt{Q/H^{5/4}}$, $Q$ in m$^3$·s$^{-1}$
- $N =$ Rotational speed, RPM
- $N_t =$ Turbine Specific-speed = $N\sqrt{P/H^{5/4}}$, $P$ in kW
- $p =$ Pressure, N·m$^{-2}$
- $pe =$ Potential energy per unit mass, J·kg$^{-1}$
- $P =$ Power developed = $\dot{m}w$, W
- $q =$ Energy exchange as heat per unit mass, J·kg$^{-1}$
- $Q =$ Total rate of energy exchange as heat or volumetric flow rate, m$^3$·s$^{-1}$
- $r =$ Radius at the point, m
- $R =$ Perfect gas constant, J·kg$^{-1}$K$^{-1}$.
- $s =$ Specific entropy, J·kg$^{-1}$K$^{-1}$.
- $T =$ Temperature, K or C
- $u =$ Internal energy, J·kg$^{-1}$ or Tangential speed of rotor = $\pi DN/60$, m·s$^{-1}$
- $v =$ Specific volume, m$^3$·kg$^{-1}$
Principles of Turbomachinery

\( V \) = Velocity of fluid, m.s\(^{-1}\)
\( w \) = Work per unit mass, J.kg\(^{-1}\)
\( x \) = Quality.
\( z \) = Height above datum, m

Greek Symbols:
\( \beta = 1 + (\gamma - 1)M^2/2 \)
\( \gamma \) = Ratio of specific heats, \( c_p/c_v \)
\( \varphi \) = Speed-ratio, \( u/V_1 \)
\( \mu \) = Dynamic viscosity, kg.m\(^{-1}\)s\(^{-1}\)
\( \Omega \) = Dimensionless Specific-speed = \( \omega \sqrt{Q/(gH)^{3/4}} \) (Pumps); = \( \omega \sqrt{(P/\rho)/(gH)^{5/4}} \) (Turbines).
\( \pi_1, \pi_2, \pi_3, \ldots \) Pi-Terms in Dimensional analysis.
\( \eta \) = Efficiency
\( \rho \) = Density of fluid, kg.m\(^{-3}\)
\( \omega \) = Angular speed, rad.s\(^{-1}\)

Subscripts:
\( a \) = Denotes adiabatic conditions
\( i,e \) = Inlet, exit
\( m \) = Model or mean,
\( o \) = Denotes stagnation/Total property
\( pa, pg \) = Power-absorbing, power-generating
\( r \) and \( s \) = Rotor and Shaft respectively,
\( 1, 2 \) = Inlet and exit respectively
\( t-t \) / \( t-s \) / \( s-t \) / \( s-s \) = Total-to-total/ Total-to-static/Static-to-total/Static-to-static

REFERENCES


**Questions and Problems**

1. Why is it the common practice to adopt a definition of efficiency based on the stagnation state for turbines and not for reciprocating engines?

2. What do you mean by a positive-displacement machine?

3. Write expressions for the total-to-static efficiencies for a turbine and a compressor and show the static and total states on a T-s diagram.

4. The dimensionless specific-speed of a turbomachine is given by Eq. (1.38b). Write the equation for specific-speed in SI units.

5. Why do we use power for turbines and flow rate for pumps in the calculation of specific-speeds?

6. Explain the importance attached to specific-speeds in turbomachines. Why do manufacturers use specific-speed as a measure of performance?

7. Why are turbomachines important these days? Explain with respect to their applications.

8. Liquid water of standard density flows at a temperature of 20°C, a static pressure of 10 bar and a velocity of 20 m·s⁻¹. Evaluate the total pressure and the total temperature of the water. [12 bar, 20°C].

9. (a) Air as a perfect gas flows in a duct at a velocity of 60 m·s⁻¹, a static pressure of two atmospheres, and a static temperature of 300 K. (a) Evaluate the total pressure and the total temperature of the air at this point in the duct. Assume ratio of specific heats to be 1.4. (b) Repeat (a) assuming the flow velocity to be 500 m·s⁻¹. [(a) 2.07 bar, 301.8 K, (b) 6.886 bar, 424.35 K].

10. The total pressure, the static pressure and the total temperature of Helium at a certain point in a duct are 10 bar, 5 bar and 400 K respectively. Assuming Helium to be a perfect gas with the ratio of specific heats 5/3, find the flow velocity in m·s⁻¹. [992.5 m·s⁻¹]
11. At the inlet to an R-12 compressor the flow Mach number is 0.8 and the total temperature 280 K. Treat R-12 as a perfect gas with a molecular weight of 120.93 and ratio of specific heats 1.10 to evaluate the inlet velocity and the ratio of total-to-static pressure. \[114.6 \text{ m.s}^{-1}, 1.41\]

12. A fluid enters a turbine at a stagnation temperature of 400 K and a stagnation pressure of 8 bar. The outlet total pressure is 1 bar. If the expansion is adiabatic and loss-free, evaluate (a) the work per kg of fluid assuming the fluid to be incompressible, with a density of 1 kg.L\(^{-1}\), (b) the work per kg of fluid assuming the fluid to be air as a perfect gas with constant properties. The change in elevation is zero. \[(a) 700 \text{ J.kg}^{-1}, (b) 179.9 \text{ kJ.kg}^{-1}\]

13. In Problem 1.12, assume that irreversibilities reduce the work by 10% of the energy released during the isentropic expansion. What is the exit fluid temperature for parts (a) and (b), assuming the expansion to remain adiabatic? For part (a), assume the specific heat is 4187 J.kg\(^{-1}\).K\(^{-1}\). \[(a) 400.017 \text{ K}, (b) 238.7 \text{ K}\]

14. Air as a perfect gas with constant properties has a total temperature of 300 K and a velocity of 300 m.s\(^{-1}\) at a certain point in a duct. What is its static temperature? What is the total pressure if the measured static pressure is 1.8 bar. \[255 \text{ K}, 3.17 \text{ bar}\]

15. Air flows through a duct of 0.1 m diameter at a rate of 2 kg.s\(^{-1}\). The measured static pressure and total temperature at a certain point in the duct are 1.1 bar and 300 K respectively. Evaluate: (a) the flow velocity, (b) the static temperature, (c) the Mach number, (d) the total pressure and (e) the density of the air. \[(a) 192 \text{ m.s}^{-1}, (b) 88^\circ\text{C}, (c) 0.567, (d) 1.37 \text{ bar}, (e) 1.35 \text{ kg.m}^{-3}\]

16. At a certain point in a duct where the air is flowing, the measured total pressure, static pressure, and total temperature are 3 bar, 2.4 bar, and 440 K respectively. Calculate: (a) the flow Mach number, (b) the static temperature, (c) the flow velocity, (d) the density (e) the flow rate per unit area. \[(a) 0.573, (b) 413 \text{ K}, (c) 233 \text{ m.s}^{-1}\]

17. Steam flows steadily at the rate of 10,000 kg.h\(^{-1}\) and produces work on the rotors at the rate of 325 kW. The inlet total pressure and temperature are 70 bar and 450°C. If the expansion is isentropic, what are the exit total pressure and total temperature? \[47 \text{ bar, 388^\circ\text{C}}\]

18. Liquid water of standard density undergoes a pressure drop of 60 bar while flowing through a constant diameter pipe. Assuming no phase change calculate the change in total temperature of the water. \[1.406^\circ\text{C}\]

19. Air as a perfect gas undergoes an increase in total pressure of 180 mm of water during its passage through a blower. The inlet static pressure is 1 bar, velocity 50 m.s\(^{-1}\) and the inlet static temperature 25°C. Evaluate the exit total pressure if the process is isentropic. Calculate the energy added by the blower in kJ.kg\(^{-1}\), if the flow is steady. If the exit velocity is 135 m.s\(^{-1}\), find also the exit static pressure and the static temperature. \[300.8 \text{ K, 1.455 kJ.kg}^{-1}, 0.911 \text{ bar, 291.8 K}\]

20. Air enters a blower at a total pressure of 1 bar, total temperature 30°C and a flow velocity of 55 m.s\(^{-1}\). The exit total temperature is 41.2°C and the velocity is 150 m.s\(^{-1}\). Calculate: (a) the change in total pressure between the inlet and the exit, and (b) change in static pressure. \[(a) 1.397 \text{ m}, (b) 150.7 \text{ mm, both of water head}\]

21. A gas with a molecular weight of 4 and ratio of specific heats 5/3 expands isentropically in a turbine through a total pressure ratio of 5 to 1. The initial total temperature is 1000 K. Find the change in total enthalpy, assuming the gas to be perfect. \[2467 \text{ kJ.kg}^{-1}\]

22. Liquid water flows through a pump with an inlet at an elevation of 1 m from the sump, to a height 2 m above the pump. The measured static pressure increases from 100 mm of Hg at the
inlet to 1.50 m at the exit of the pump. The inlet and the exit velocities in the pipes are 5 and 10 m·s⁻¹ respectively. Calculate the isentropic change in total enthalpy across the device. [253.6 J·kg⁻¹]

23. Air enters a compressor with a static pressure of 15 bar, static temperature 15°C and a velocity of 50 m·s⁻¹. At the exit, the static pressure is 30 bar, temperature 100°C and velocity 100 m·s⁻¹. The outlet is 1 metre above the inlet. Calculate the isentropic change in total enthalpy, and the actual change in total enthalpy. [67.1 kJ·kg⁻¹, 89.1 kJ·kg⁻¹]

24. A fluid flowing through a turbine has:

<table>
<thead>
<tr>
<th>Inlet</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity = 3 m·s⁻¹</td>
<td>Velocity = 6 m·s⁻¹</td>
</tr>
<tr>
<td>Pressure, static = 10 bar</td>
<td>Pressure, static = 2 bar</td>
</tr>
<tr>
<td>Elevation = 2 m</td>
<td>Elevation = 1.8 m.</td>
</tr>
</tbody>
</table>

Find the isentropic change in total enthalpy between the inlet and the exit of the turbine if the fluid is: (a) water, (b) oil with a specific gravity of 0.8, and (c) mercury with a specific gravity of 13.6. If the fluid is air with an inlet temperature of 30°C, compare the change in total enthalpy of the air with that of the other fluids. Which of the following quantities is significant in the four cases above: (i) Static enthalpy change, (ii) kinetic energy change and, (iii) potential energy change? [788.5 J·kg⁻¹, 988 J·kg⁻¹, 47.28 J·kg⁻¹].

25. Consider a steady flow process and identify correctly for the following processes whether the total pressure and temperature increase, decrease or remain constant. Ignore changes in elevation.

| A | Reversible, adiabatic, Incompress., work-free |
| B | Irreversible, adiabatic, Incompressible, work-free. |
| C | Reversible, adiabatic, perfect gas and work-free |
| D | Reversible, adiabatic, perfect gas, with work done on gas |
| E | Reversible, adiabatic, Incompressible, work done by fluid |
| F | Reversible, adiabatic, Incompressible, work-free |
| G | Irreversible, adiabatic, perfect gas work done by gas. |

26. The total-to-total efficiency of a power-absorbing machine handling liquid water of standard density is 70%. Suppose the total pressure of the water is increased by 4 bar. Find: (a) the isentropic change in total enthalpy, (b) the actual change in total enthalpy, (c) the change in temperature of the water and, (d) the power input if the flow rate is 30 kg·s⁻¹. [(a) 400 kJ·kg⁻¹, (b) 571.4 kJ·kg⁻¹, (c) 0.04°C (d) 21.52 kW]
27. A power-generating turbomachine generates 100 kW when the flow through the device is 0.1 m$^3$s$^{-1}$ of oil with a specific gravity of 0.8. The total-to-total efficiency is 75%. Find (a) the change in total pressure of the oil, (b) the change in static pressure if the inlet and exit velocities are 3 and 10 m$\cdot$s$^{-1}$ respectively. [-13.3 bar, -13.7 bar]

28. Air flows through a blower where its total pressure is increased by 150 mm of water. At the inlet, $p_{in} = 1.05$ bar and $T_{in} = 15°C$. The total-to-total efficiency is 70%. Find the exit total pressure, the exit total temperature, the isentropic and actual changes in total enthalpy. [1.065 bar, 16.2°C, 1.205 kJ$\cdot$kg$^{-1}$ and 1.721 kJ$\cdot$kg$^{-1}$].

29. Hot gases at a total pressure of 2 bar and total temperature 1000 K, enter a gas turbine at the exit of which the total pressure and temperature are 1.5 bar and 830 K. If the specific-heat ratio is 1.3, the molecular weight is 28.7 and the exit velocity is 250 m$\cdot$s$^{-1}$, calculate the total-to-total and total-to-static efficiencies. [82.1%, 76.3%]

30. Oil with a specific gravity of 0.75 enters an axial pump at a total pressure of 2 bar and a flow-rate of 100 kg$\cdot$s$^{-1}$. The input power is 20 kW and the total-to-total efficiency is 0.7. The flow is in a horizontal plane. Find: the exit static pressure if the exit area is 0.02 m$^2$, the change in internal energy in J$\cdot$s$^{-1}$, and the change in total pressure that would occur if the efficiency of the turbine is 100%. [2.14 bar, 6000 J$\cdot$s$^{-1}$, 1.50 bar]

31. Water enters a hydraulic turbine with a static pressure of 3 bar and a velocity of 5 m$\cdot$s$^{-1}$. At the exit, the velocity is 10 m$\cdot$s$^{-1}$ and the elevation is 3 m below the inlet. The flow area in the outlet pipe increases by a ratio of 5:1 at which point it opens to the atmosphere. Assuming no frictional losses or change in elevation of the exit pipe, find the total-to-total efficiency if the total-to-static efficiency is 78%. [82.1%, 76.3%]

32. The initial and final total pressures of a fluid are 1 bar and 10 bar. The initial total temperature is 10°C. Find the steady flow work of compression, if the fluid is water and the total-to-total efficiency is 75%. What will it be if the fluid is air? [1200 J$\cdot$kg$^{-1}$ and 353 kJ$\cdot$kg$^{-1}$]

33. The change in total enthalpy of fluid flowing through a turbomachine is 6.1 kJ$\cdot$kg$^{-1}$ when the inlet total pressure and temperature are 1.013 bar and 30°C respectively. What class of turbomachine is this? Find the exit total temperature if the fluid is air. If the total-to-total efficiency is 75% and the fluid is air, what is the total pressure ratio across the machine? Also find the total pressure ratio across the machine if the fluid is water and the efficiency is the same as with air. [Power-absorbing, 36.1°C, 1.054 and 47.1]

34. A model operates under a head of 5 m at 1200 RPM. The power in the laboratory is limited to 8 kW. Predict the power and the diameter-ratio of a prototype turbine which operates under a head of 40 m at 240 RPM. What type of turbine is the prototype? Pelton wheel, Francis or Kaplan? [36.2 MW, $D/d_m = 14.14$, Kaplan turbine]

35. The power output $P$, of a windmill is a function of the windmill diameter, $D$, the angular speed of rotation $\omega$, number of blades $Z$, as well as the velocity and density of the air, $V$ and $\rho$, respectively. Carry out a dimensional analysis and exhibit a non-dimensional equation showing the dependence of $P$ on various parameters. [$P/(\rho \omega^3 D^5), V/(D \omega Z)$].

36. A small windmill model, 0.55 m in diameter develops 4.5 kW at sea-level ($T = 25°C$) when the air velocity is 40 m$\cdot$s$^{-1}$ and the rotational speed is 500 rad$\cdot$s$^{-1}$. Find the power that a dynamically and geometrically similar windmill of diameter 5.5 m develops at a wind speed of 20 m$\cdot$s$^{-1}$ and an
altitude of 1800 m above sea-level. Assume that the barometric pressure of air varies with altitude as provided by the equation: \( h_b = 10 - \frac{h}{900} \), where \( h \) = altitude in m, above sea-level and \( h_b \) = barometric pressure in m of water at altitude \( h \).

37. The following data refer to a model and the prototype:

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Prototype</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head, m</td>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>Discharge m(^3).s(^{-1})</td>
<td>0.12</td>
<td>60</td>
</tr>
<tr>
<td>RPM</td>
<td>1200</td>
<td>300</td>
</tr>
<tr>
<td>Power, kW</td>
<td>6</td>
<td>?</td>
</tr>
</tbody>
</table>

Find the diameter-ratio of prototype to model, power, specific-speed (conventional) and dimensionless. (12.65, 32.75 MW, 325.1 and 2.032)

38. A prototype Pelton wheel is to operate under a head of 400 m at 180 RPM. A model of the prototype develops 8 kW under a head of 30 m at 420 RPM. It uses 1 m\(^3\).s\(^{-1}\) of water. Find for the prototype, the specific-speed, the scale-ratio, power and water discharge. Use appropriate scaling equations as required. [16.9, 8.5, 28.2 MW]

39. A hydraulic turbine develops 60 MW under a head of 12.5 m. A model is fabricated for this turbine to work under a head of 2 m and at 240 RPM develops 60 kW when water flow rate is 3800 kg.s\(^{-1}\). Determine the type of turbine required for the prototype. Also find the speed, scale-ratio, volumetric flow rate of water and the power output. [Kaplan turbine, 75 RPM, 8.2, 638.9 m\(^3\).s\(^{-1}\) and 1357.6 kW].