Introduction to Engineering Mechanics

The state of rest and state of motion of the bodies under the action of different forces has engaged the attention of philosophers, mathematicians and scientists for many centuries. The branch of physical science that deals with the state of rest or the state of motion is termed as **Mechanics**. Starting from the analysis of rigid bodies under gravitational force and simple applied forces the mechanics has grown to the analysis of robotics, aircrafts, spacecrafts under dynamic forces, atmospheric forces, temperature forces etc.

Archimedes (287–212 BC), Galileo (1564–1642), Sir Issac Newton (1642–1727) and Einstein (1878–1955) have contributed a lot to the development of mechanics. Contributions by Varignon, Euler, D. Alembert are also substantial. The mechanics developed by these researchers may be grouped as

(i) Classical mechanics/Newtonian mechanics
(ii) Relativistic mechanics
(iii) Quantum mechanics/Wave mechanics.

Sir Issac Newton, the principal architect of mechanics, consolidated the philosophy and experimental findings developed around the state of rest and state of motion of the bodies and put forth them in the form of three laws of motion as well as the law of gravitation. The mechanics based on these laws is called **Classical mechanics** or **Newtonian Mechanics**.

Albert Einstein proved that Newtonian mechanics fails to explain the behaviour of high speed (speed of light) bodies. He put forth the theory of **Relativistic Mechanics**.

Schrödinger (1887–1961) and Broglie (1892–1965) showed that Newtonian mechanics fails to explain the behaviour of particles when atomic distances are concerned. They put forth the theory of **Quantum Mechanics**.

Engineers are keen to use the laws of mechanics to actual field problems. Application of laws of mechanics to field problem is termed as **Engineering Mechanics**. For all the problems between atomic distances to high speed distances Classical/Newtonian mechanics has stood the test of time and hence that is the mechanics used by engineers. Therefore in this text classical mechanics is used for the analysis of engineering problems.

### 1.1 CLASSIFICATION OF ENGINEERING MECHANICS

Depending upon the body to which the mechanics is applied, the engineering mechanics is classified as

(a) Mechanics of Solids, and
(b) Mechanics of Fluids.

The solid mechanics is further classified as mechanics of rigid bodies and mechanics of deformable bodies. The body which will not deform or the body in which deformation can be neglected in the analysis, are called as **Rigid Bodies**. The mechanics of the rigid bodies dealing with the bodies at rest is termed as **Statics** and that dealing with bodies in motion is called
**Dynamics.** The dynamics dealing with the problems without referring to the forces causing the motion of the body is termed as Kinematics and if it deals with the forces causing motion also, is called Kinetics.

If the internal stresses developed in a body are to be studied, the deformation of the body should be considered. This field of mechanics is called Mechanics of Deformable Bodies/Strength of Materials/Solid Mechanics. This field may be further divided into Theory of Elasticity and Theory of Plasticity.

Liquid and gases deform continuously with application of very small shear forces. Such materials are called Fluids. The mechanics dealing with behaviour of such materials is called Fluid Mechanics. Mechanics of ideal fluids, mechanics of viscous fluids and mechanics of incompressible fluids are further classification in this area. The classification of mechanics is summarised below in flowchart.

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**1.2 BASIC TERMINOLOGIES IN MECHANICS**

The following are the basic terms to study mechanics, which should be understood clearly:

**Mass**

The quantity of the matter possessed by a body is called mass. The mass of a body will not change unless the body is damaged and part of it is physically separated. When a body is taken out in a spacecraft, the mass will not change but its weight may change due to change in gravitational force. Even the body may become weightless when gravitational force vanishes but the mass remains the same.

**Time**

Time is the measure of succession of events. The successive event selected is the rotation of earth about its own axis and this is called a day. To have convenient units for various activities, a day is divided into 24 hours, an hour into 60 minutes and a minute into 60 seconds. Clocks are the instruments developed to measure time. To overcome difficulties due to irregularities in the earth’s rotation, the unit of time is taken as second which is defined as the duration of 9192631770 period of radiation of the cesium-133 atom.
Space

The geometric region in which study of body is involved is called space. A point in the space may be referred with respect to a predetermined point by a set of linear and angular measurements. The reference point is called the origin and set of measurements as 'coordinates'. If coordinates involve only in mutually perpendicular directions, they are known as Cartesian coordinates. If the coordinates involve angle and distances, it is termed as polar coordinate system.

Length

It is a concept to measure linear distances. The diameter of a cylinder may be 300 mm, the height of a building may be 15 m. Actually metre is the unit of length. However, depending upon the sizes involved micro, milli or kilo metre units are used for measurement. A metre is defined as length of the standard bar of platinum-iridium kept at the International Bureau of Weights and Measures. To overcome difficulties of accessibility and reproduction, now metre is defined as 1690763.73 wavelength of krypton-86 atom.

Displacement

Displacement is defined as the distance moved by a body/particle in the specified direction. Referring to Fig. 1.1, if a body moves from position A to position B in the x-y plane shown, its displacement in x-direction is \( AB' \) and its displacement in y-direction is \( BB' \).

Velocity

The rate of change of displacement with respect to time is defined as velocity.

Acceleration

Acceleration is the rate of change of velocity with respect to time. Thus

\[
a = \frac{dv}{dt}, \text{ where } v \text{ is velocity}
\]

\[\text{(1.1)}\]

Momentum

The product of mass and velocity is called momentum. Thus

\[
\text{Momentum} = \text{Mass} \times \text{Velocity}
\]

\[\text{(1.2)}\]

Continuum

A body consists of several matters. It is a well known fact that each particle can be subdivided into molecules, atoms and electrons. It is not possible to solve any engineering problem by treating a body as a conglomeration of such discrete particles. The body is assumed to consist of a continuous distribution of matter. In other words, the body is treated as continuum.

Rigid Body

A body is said to be rigid, if the relative positions of any two particles in it do not change under the action of the forces. In Fig. 1.2 (a), points A and B are the original positions of two
points of a body. After application of a system of forces \( F_1, F_2, F_3 \), the body takes the position as shown in Fig. 1.2 (b). \( A' \) and \( B' \) are the new positions of \( A \) and \( B \). If the body is treated as rigid, the relative position of \( A'B' \) and \( AB \) are the same i.e.,

\[ A'B' = AB. \]

\[ \text{Fig. 1.2} \]

Many engineering problems can be solved satisfactorily by assuming bodies rigid bodies.

**Particle**

A particle may be defined as an object which has only mass and no size. Such a body cannot exist theoretically. However, in dealing with problems involving distances considerably larger compared to the size of the body, the body may be treated as particle, without sacrificing accuracy. Examples of such situations are

— A bomber aeroplane is a particle for a gunner, operating from the ground.
— A ship in mid sea is a particle in the study of its relative motion from a control tower.
— In the study of movement of the earth in celestial sphere, earth is treated as a particle.

### 1.3 LAWS OF MECHANICS

The following are the fundamental laws of mechanics:

- Newton's first law,
- Newton's second law,
- Newton's third law,
- Newton's law of gravitation,
- Law of transmissibility of forces, and
- Parallelogram law of forces.

**Newton’s First Law**

*It states that every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external agency acting on it.* This leads to the definition of force as the external agency which changes or tends to change the state of rest or uniform linear motion of the body.

**Newton’s Second Law**

*It states that the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it.* Thus according to this law,
Force \( \propto \) Rate of change of momentum. But momentum = Mass \( \times \) Velocity

As mass do not change,

\[
\text{Force} \propto \text{Mass} \times \text{Rate of change of velocity}
\]

\[i.e., \quad \text{Force} \propto \text{Mass} \times \text{Acceleration}
\]

\[ F = m \times a \quad \ldots \quad (1.3) \]

**Newton's Third Law**

*It states that for every action there is an equal and opposite reaction.* Consider the two bodies in contact with each other. Let one body applies a force \( F \) on another. According to this law, the second body develops a reactive force \( R \) which is equal in magnitude to force \( F \) and acts in the line same as \( F \) but in the opposite direction. Figure 1.3 shows the action of the ball and the reaction from the floor. In Fig. 1.4, the action of the ladder on the wall and the floor and the reactions from the wall and floor are shown.

![Fig. 1.3](image)

**Newton's Law of Gravitation**

Everybody attracts the other body. The force of attraction between any two bodies is directly proportional to their masses and inversely proportional to the square of the distance between them. According to this law, the force of attraction between the bodies of mass \( m_1 \) and mass \( m_2 \) at a distance \( d \) as shown in Fig. 1.5 is

\[
F = G \frac{m_1 m_2}{d^2} \quad \ldots \quad (1.4)
\]

where \( G \) is the constant of proportionality and is known as constant of gravitation.
Law of Transmissibility of Force

According to this law, the state of rest or motion of the rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.

Let $F$ be the force acting on a rigid body at point $A$ as shown in Fig. 1.6. According to the law of transmissibility of force, this force has the same effect on the state of body as the force $F$ applied at point $B$.

In using law of transmissibility of forces, it should be carefully noted that it is applicable only if the body can be treated as rigid. In this text, the engineering mechanics is restricted to study of state of rigid bodies and hence this law is frequently used. Same thing cannot be done in the subject ‘solid mechanics’ where the bodies are treated as deformable and internal forces in the body are studied.

The law of transmissibility of forces can be proved using the law of superposition, which can be stated as the action of a given system of forces on a rigid body is not changed by adding or subtracting another system of forces in equilibrium.

Consider the rigid body shown in Fig. 1.7 (a). It is subjected to a force $F$ at $A$. $B$ is another point on the line of action of the force. From the law of superposition, it is obvious that if two equal and opposite forces of magnitude $F$ are applied at $B$ along the line of action of given force $F$, [Ref. Fig. 1.7 (b)] the effect of given force on the body is not altered. Force $F$ at $A$ and opposite force $F$ at $B$ form a system of forces in equilibrium. If these two forces are subtracted from the system, the resulting system is as shown in Fig. 1.7 (c). Looking at the system of forces in Figs. 1.7 (a) and 1.7 (c), we can conclude the law of transmissibility of forces is proved.
Parallelogram Law of Forces

The parallelogram law of forces enables us to determine the single force called resultant which can replace the two forces acting at a point with the same effect as that of the two forces. This law was formulated based on experimental results. Though, Stevinces employed it in 1586, the credit of presenting it as a law goes to Varignon and Newton (1687). This law states that, if two forces acting simultaneously on a body at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces.

In Fig. 1.8, the force $F_1 = 4$ units and force $F_2 = 3$ units are acting on a body at point $A$. Then to get resultant of these forces, parallelogram $ABDC$ is constructed such that $AB$ is equal to $4$ units to linear scale and $AC$ is equal to $3$ units. Then according to this law, the diagonal $AD$ represents the resultant in the direction and magnitude. Thus the resultant of the forces $F_1$ and $F_2$ on the body is equal to units corresponding to $AD$ in the direction $\alpha$ to $F_1$.

![Fig. 1.8](image)

1.4 DERIVED LAWS

Referring to Fig. 1.8 (b), we can get the resultant $AD$ by constructing triangle $ABD$. Line $AB$ is drawn to represent $F_1$ and $BD$ to represent $F_2$. Then $AD$ should represent the resultant of $F_1$ and $F_2$. Thus, we have derived triangle law of forces from fundamental law, parallelogram law of forces. The **Triangle Law of Forces** may be stated as If two forces acting on a body are represented one after another by the sides of a triangle, their resultant is represented by the closing side of the triangle taken from first point to the last point.

If more than two concurrent forces are acting on a body, two forces at a time can be combined by triangle law of forces and finally resultant of all the forces acting on the body may be obtained.

A system of 4 concurrent forces acting on a body are shown in Fig. 1.9. $AB$ represents $F_1$ and $BC$ represents $F_2$. Hence, according to triangle law of forces $AC$ represents the resultant of $F_1$ and $F_2$, say, $R_1$. 
If \( CD \) is drawn to represent \( F_3 \), then from triangle law of forces \( AD \) represents, the resultant of \( R_1 \) and \( F_3 \). In other words, \( AD \) represents the resultant of \( F_1 \), \( F_2 \) and \( F_3 \). Let it be called as \( R_2 \).

On the same line, logic can be extended to say that \( AE \) represents the resultant of \( F_1 \), \( F_2 \), \( F_3 \) and \( F_4 \) if \( DE \) represents \( F_4 \). Thus resultant \( R \) is represented by the closing line of the polygon \( ABCDE \) in the direction \( AE \). Thus we have derived **Polygon Law of Forces** and it may be stated as ‘If a number of concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in a order, then the resultant is represented in magnitude and direction by the closing side of the polygon, taken from first point to last point’.

### 1.5 Units

Length \((L)\), Mass \((M)\) and Time \((S)\) are the fundamental units in mechanics. The units of all other quantities may be expressed in terms of these basic units. The three commonly used systems in engineering are

- Metre—Kilogramme—Second (MKS) system,
- Centimetre—Gramme—Second (CGS) system, and
- Foot—Pound—Second (FPS) system.

The units of length, mass and time used in the system are used to name the systems. Using these basic units, the units for other quantities can be found. For example, in MKS the units for the various quantities are as shown below:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>Square metre</td>
<td>(m^2)</td>
</tr>
<tr>
<td>Volume</td>
<td>Cubic metre</td>
<td>(m^3)</td>
</tr>
<tr>
<td>Velocity</td>
<td>Metre per second</td>
<td>(m/sec)</td>
</tr>
<tr>
<td>Acceleration</td>
<td>Metre per second per second</td>
<td>(m/sec^2)</td>
</tr>
</tbody>
</table>

**Unit of Force**

Presently the whole world is in the process of switching over to SI system of units. SI stands for System Internationale d’ units or International System of units. As in MKS system, in SI system also the fundamental units are metre for length, kilogramme for mass and second
for time. The difference between MKS and SI system arise mainly in selecting the unit of force. From Eqn. (1.3), we have

\[ \text{Force} \propto \text{Mass} \times \text{Acceleration} = k \times \text{Mass} \times \text{Acceleration} \quad \ldots(1.5) \]

In SI system, the unit of force is defined as the force which causes 1 kg mass to move with an acceleration of 1 m/sec\(^2\) and is termed as 1 Newton. Hence, the constant of proportionality \( k \) becomes unity. Unit of force can be derived from Eqn. (1.5) as

\[ \text{Unit of Force} = \text{kg} \times \text{m/sec}^2 = \text{kg-m/sec}^2 \]

In MKS, the unit of force is defined as the force which makes a mass of 1 kg to move with gravitational acceleration \( g \) m/sec\(^2\). This unit of force is called kilogramme weight or kg-wt. Gravitational acceleration is 9.81 m/sec\(^2\) near the earth surface. In all the problems encountered in engineering mechanics the variation in gravitational acceleration is negligible and may be taken as 9.81 m/sec\(^2\). Hence, the constant of proportionality in Eqn. (1.5) is 9.81, which means

\[ 1 \text{ kg-wt} = 9.81 \text{ newton} \quad \ldots(1.6) \]

It may be noted that in public usage, kg-wt force is called as kg only.

**Unit of Constant of Gravitation**

From Eqn. (1.4),

\[ F = G \frac{m_1 m_2}{d^2} \quad \text{or} \quad G = \frac{F d^2}{m_1 m_2} \]

\[ \therefore \quad \text{Unit of } G = \frac{N \times \text{m}^2}{\text{kg} \times \text{kg}} = \text{Nm}^2/\text{kg}^2 \]

It has been proved by experimental results that the value of \( G = 6.673 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \). Thus, if two bodies, one of mass 10 kg and the other of 5 kg are at a distance of 1 m, they exert a force

\[ F = \frac{6.673 \times 10^{-11} \times 10 \times 5}{1^2} = 33.365 \times 10^{-10} \text{ N} \]
on each other.

Now let us find the force acting between 1 kg-mass near earth surface and the earth. Earth has a radius of \( 6371 \times 10^3 \) m and has a mass \( 5.96506 \times 10^{24} \) kg. Hence the force between the two bodies is

\[ F = \frac{6.673 \times 10^{-11} \times 1 \times 5.96504 \times 10^{24}}{(6371 \times 10^3)^2} = 9.80665 \text{ N}. \]

In common usage, we call the force exerted by earth on a body as weight of the body. Thus weight of 1 kg mass on earth surface is 9.80665 N, which is approximated as 9.81 N for all practical problems. Compared to this force, the force exerted by two bodies near earth surface is negligible as may be seen from the example of 10 kg and 5 kg mass bodies.

Denoting the weight of the body by \( W \), from Eqn. (1.4), we get

\[ W = \frac{G m M_e}{r^2} \]

where \( m \) is the mass of the body,

\( M_e \) is the mass of the earth, and

\( r \) is the radius of the earth.
Denoting $\frac{GM_e}{r^2}$ by $g$, we get

$$W = mg = 9.81 \, \text{m}$$

...(1.7)

Unit of $g$ can be obtained as follows:

$$g = \frac{GM_e}{r^2}$$

Unit of $g = \frac{\text{Nm}^2}{(\text{kg})^2} \times \frac{\text{kg}}{\text{m}^2} = \frac{\text{N}}{\text{kg}}$

as unit of Newton force is kg-m/sec$^2$, we get

Unit of $g = \frac{\text{kg} \cdot \text{m/sec}^2}{\text{kg}} = \text{m/sec}^2$

Hence, $g$ may be called as acceleration due to gravity. Any body falling freely near earth surface experiences this acceleration. The value of $g$ is 9.81 m/sec$^2$ near the earth surface as can be seen from Eqn. (1.7).

The prefixes used in SI system when quantities are too big or too small are shown in Table 1.1.

<table>
<thead>
<tr>
<th>Multiplying Factor</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{12}$</td>
<td>tera</td>
<td>T</td>
</tr>
<tr>
<td>$10^{9}$</td>
<td>giga</td>
<td>G</td>
</tr>
<tr>
<td>$10^{6}$</td>
<td>mega</td>
<td>M</td>
</tr>
<tr>
<td>$10^{3}$</td>
<td>kilo</td>
<td>k</td>
</tr>
<tr>
<td>$10^{0}$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>milli</td>
<td>m</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>micro</td>
<td>μ</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>nano</td>
<td>n</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>pico</td>
<td>p</td>
</tr>
<tr>
<td>$10^{-15}$</td>
<td>femto</td>
<td>f</td>
</tr>
<tr>
<td>$10^{-18}$</td>
<td>atto</td>
<td>a</td>
</tr>
</tbody>
</table>

### 1.6 CHARACTERISTICS OF A FORCE

From Newton’s first law, we defined the force as the agency which tries to change state of stress or state of uniform motion of the body. From Newton’s second law of motion we arrived at practical definition of unit force as the force required to produce unit acceleration in a body of unit mass. Thus 1 newton is the force required to produce an acceleration of 1 m/sec$^2$ in a body of 1 kg mass. It may be noted that a force is completely specified only when the following four characteristics are specified:

— Magnitude,
— Point of application,
— Line of action, and
— Direction.
In Fig. 1.10, \( AB \) is a ladder kept against a wall. At point \( C \), a person weighing 600 N is standing. The force applied by the person on the ladder has the following characters:

- magnitude is 600 N,
- the point of application is at \( C \) which is 2 m from \( A \) along the ladder,
- the line of action is vertical, and
- the direction is downward.

Note that, the magnitude of the force is written near the arrow. The line of the arrow shows the line of application and the arrow head represents the point of application and the direction of the force.

### 1.7 SYSTEM OF FORCES

When several forces act simultaneously on a body, they constitute a system of forces. If all the forces in a system do not lie in a single plane they constitute the system of forces in space. If all the forces in a system lie in a single plane, it is called a coplanar force system. If the line of action of all the forces in a system pass through a single point, it is called a concurrent force system. In a system of parallel forces all the forces are parallel to each other. If the line of action of all the forces lie along a single line then it is called a collinear force system. Various system of forces, their characteristics and examples are given in Table 1.2 and shown in Fig. 1.11.

<table>
<thead>
<tr>
<th>Force System</th>
<th>Characteristics</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collinear forces</td>
<td>Line of action of all the forces act along the same line.</td>
<td>Forces on a rope in a tug of war.</td>
</tr>
<tr>
<td>Coplanar parallel forces</td>
<td>All forces are parallel to each other and lie in a single plane.</td>
<td>System of forces acting on a beam subjected to vertical loads (including reactions).</td>
</tr>
<tr>
<td>Coplanar like parallel forces</td>
<td>All forces are parallel to each other, lie in a single plane and are acting in the same direction.</td>
<td>Weight of a stationary loads on a rail when the track is straight.</td>
</tr>
<tr>
<td>Coplanar concurrent forces</td>
<td>Line of action of all forces pass through a single point and forces lie in the same plane.</td>
<td>Forces on a rod resting against a wall.</td>
</tr>
<tr>
<td>Coplanar non-concurrent forces</td>
<td>All forces do not meet at a point, but lie in a single plane.</td>
<td>Forces on a ladder resting against a wall when a person stands on a rung which is not at its centre of gravity.</td>
</tr>
<tr>
<td>Non-coplanar parallel forces</td>
<td>All the forces are parallel to each other, but not in same plane.</td>
<td>The weight of benches in a classroom.</td>
</tr>
<tr>
<td>Non-coplanar concurrent forces</td>
<td>All forces do not lie in the same plane, but their lines of action pass through a single point.</td>
<td>A tripod carrying a camera.</td>
</tr>
<tr>
<td>Non-coplanar non-concurrent forces</td>
<td>All forces do not lie in the same plane and their lines of action do not pass through a single point.</td>
<td>Forces acting on a moving bus.</td>
</tr>
</tbody>
</table>